Talmudic Metrology IX: Back to the Talmudic Units of Measure of Volume and Capacity.

1. Introduction.

In a former paper headed: Talmudic Metrology III: Units of Measure of Volume and Capacity,[1] we analysed the problematic of the Talmudic units of measure of volume and capacity and especially the conundrum raised from the 15th century onward and again with more acuity in the second half of the eightteenth century, because of the contradiction between two contradictory methods of evaluation ot the units of capacity, the evaluation of the volume of the capacities determined by the breadth of thumb and that determined by the volume of the eggs.
In lack of more objective data, the use of the natural values of the two following data, the average breadth of thumbs from one side and the use of the average volume of the hen eggs leads to an internal contradiction. Indeed the natural average breadth of thumbs is estimated to 2.4 – 2.5 cm and the average volume of an henn egg is about 50 cm3. The fundamental formula binding the units of length and volume is the following:
1.5 eggs = 1 Rev'it = 10.8 E^3, where E is Etsba and represents the breadth of thumb.
If we introduce E= 2.4 cm we get 1.5 eggs = 1 Revi'it = 149.29 cm^3 and 1 egg = 99.53 cm^3.
If we introduce egg = 50 cm^3 we get 75 = 10.8 E^3 and E = 1.9079 cm.
From the end of the 18th century onwards, we had two main schools : those championing the small units of capacity, based on the volume of the natural hen egg and accepting the consequences on the units of length and those championing the large units of volume based on the average breadth of thumb, despite the consequences on the volume of the eggs. The latters were obliged to accept the principle that originally the eggs were greater and their volume later diminished in the course of history. A third small group wanted, in order to solve the old conundrum, to allow the coexistence between small units of capacity and bigger and normal units of length by considering a theoretical Mikveh of semi-elipsoïdal shape, with a circuar basis having a diameter of 1 Cubit and with a height of 3 Cubits. The first group followed the metrology of Maimonides or that of Rabbi Naé, while the second group followed the metrology of Hazon Ish.
In a former paper headed: Talmudical Metrology I: The Mile as a Unit of Length,[2] we adopted a new point of view: we developped a new theory about the units of length according which the Talmudic units of length are not based on the average breadth of thumbs of people but it has an absolute definition. We proved on the basis of different Talmudic quotations referring to controllable distances between known places, expressed in miles that the Talmudic Mile is exactly the Roman Mile. Furthermore we noted that the tehum Shabbat[3] is generally considered as a Mile and finally according to many

[1] BDD 21, March 2009, editor Ely Merzbach, Bar Ilan University Press pp. 7 – 59. The present paper complements the former paper. An exhaustive bibliography can be found on pages 52 – 53 of that paper.
[2] BDD 19, January 2008, pp. 55 – 83.
[3] The walkable distance on Shabbat.

authorities the tehum Shabbat is 2828.43 Cubits, hence the diagonale of a square whose sides have a length of 2000 Cubits. Therefore it seems likely that the Roman Mile is the diagonale of a square whose sides are 2000 Cubits. According to the last estimation the length of the Roman Mile is 1481.5m,[4] and therfore the Talmudic Cubit is 52.38 cm and Etsba,[5] the breadth of the thumb is 2.1825 cm.
These Talmudic units of length depend thus on the Roman mile, which the Talmudic Rabbis adopted for their mile. As long as the Jewish people lived under Roman rule, this situation was evident. But later, when the main part of Judaism lived under Persian rule, this was forgotten. Our assumption about the Talmudic Cubit is confirmed by archeological findings. The examination of the archeological remnants of the Temple and the Temple Mount and the comparison with the measurements given in the Mishna Midot represent another way to determine the Talmudic units of Length. Asher Grosberg has noted that the Cubit which best fits the agreement between the archeological remains and the descriptions and measures from Mishna Midot and Josephus, is a Cubit of about 52.5 cm.[6] Other archeological remains confirm this order of size of the Talmudic cubit. Daniel Levi summarized various archeological data confirming that the Cubit ranged between 52 and 52.5 cm in a paper published in Tehumin 30.[7]

In the first above mentioned paper we analysed several Talmudic passages proving that the Talmudic units of measure emulated the corresponding Roman units of measure, whether we deal with the units of length, of weight or of capacity. Different Talmudic quotations going back to the period of the Mishna, give evidence of this fact, which perhaps, was a real adaptation of ancient units of measure slightly different in order of copying the Roman units of measure. This is certainly the case for the units of measure of weight as the following expression shows :

כיכר דבלה של שישים מנה באיטלקי, שביעית א׳, ב׳ ו א׳, ג׳.

This is also the case for the units of measure of capacity as the following expression show.

מדות הלח והיבש, שיעורן באיטלקי, זו מדברית, כלים י״ז: י״א.
חצי לוג יין באיטלקי, משנה סנהדרין ח׳: ב׳, ירושלמי סנהדרין מ״א ע״ב (דפוס וילנא).
וכולם במדה האיטלקית, תוספתא כתובות ה׳: ז׳.
רביעית יין באיטלקית, עירובין ס״ד ע״ב, ירושלמי עבודה זרה פ״א ה״ט, ויקרא רבה ל״ז.

The first quotation above-mentioned shows that they considered the new Italian measures as the measures of the desert, thus the units used by Moshe Rabbaeinu.[8] Clearly the standard of the Talmudic units of measure of volume and capacity was similar to the Roman units of capacity. Similar but not identical because the proportion between

[4] According to the accessible data the length of the Roman mile ranges between 1473m and 1482m with a preference in the 19th century for 1478m and today for 1481.5m : In this paper we adopted 1481.5m. But it should be remembered that there is a little degree of incertitude.
[5] The cubit is equal to 6 Tefah or handbreadth and to 6* 4= 24 etsba or thumb.
[6] Asher Grosberg: Ittur Tehumei har ha-Bayit u Makom ha-Mikdash, Tehumin 5756.
[7] Daniel Moshe Levi : Pitron hidat ha-shiourim al-pi ha-archeologia, Tehumin 30, 5770, pp. 420 – 421.
[8] See the commentary of R. Samson ben Avraham of Sens on Mishnah Shevi'it I: 2.

the sequence of the consecutive Talmudic and Roman units of capacity is not the same. This lets an area of incertitude in the attribution of the Roman correspondants of the Talmudic units of capacity. The comparison of both sequences of units of capacity lets a doubt remain in the Roman attribution of the Log: Sextarius of 0.547 liters or Hemina of 0.274 liters. This hesitation and even confusion is confirmed and amplified in the extant translations of the Bible by the Septuaginta and Jerome and testimonies of Josephus. The choice of the first solution would give us the solution of the large Talmudic units of capacity and the choice of the second solution would give us the solution of the small Talmudic units of capacity. We based ourselves on different Talmudic passages which seem to accredit the theory that the Talmud followed the theory of the large units of capacity. We quote below two Talmudic passages supporting this thesis. The first following quotation supports the principle that Log corresponds to Kestes, the Greek denomination parallel to the Roman Sextarius. The second quotation seems to indicate that the Talmud identifies the Revi'it with the Quartarius, the fourth part of the Sextarius.

קסתא דמורייסא דהווה בציפורי היא הוות כמין לוגא דמקדשא ובה משערין רביעית של פסח, פסחים ק״ט ע״א.
רביעית יין באיטלקית, עירובין ס״ד ע״ב, ירושלמי עבודה זרה פ״א ה״ט, ויקרא רבה ל״ז.

Therefore we considered in the above mentioned paper the principle that the Talmudic standard of units of measure was based on the units of length derived from the Roman Mile giving a Cubit of 52.38 cm and an Etsba of 2.1825 cm from one side and on units of capacity derived from the relation 1 Log = 1 Sextarius = 547 cm^3.
If we submit our new standard of unity of measure of volumes, based on a Etsba of 2.1825 cm and a Log of 0.547 liters to the test of the fundamental formula[9] binding the units of length and volume:

1.5 eggs = 1 Revi'it = 10.8 E^3,

we find : $1.5 * 91.1667 = (547 / 4) = 136.75 = 10.8 * (2.1825)^3 = 112.2757$ cm^3.
Thus the standard achieved, which should fulfill the Talmudic requirements and ensure a correct Cubit and a correct capacity for the Log, does not satisfy the fundamental formula binding the units of length and volume. Furthermore the volume of the average egg of hen corresponding to this standard of measure is 91.1667 cm^3 , which is nearly twice the volume of an average egg.
In order to solve the conundrum resulting from the contradiction between these large Talmudic units of capacity and the units of length we proposed to consider that the volume of the eggs considered is not the volume of the egg stricto sensu but the volume of the parallelepiped circumscribed to the egg. According to this conception, a volume of 144 eggs would not mean that it represents the volume of water displaced by this number of eggs but it would mean a volume in which it is possible to store 144 eggs of an average size. Therefore a volume of 91.1667 cm^3 for an egg corresponds to an real egg of 47.73 cm^3.[10]
What about the problem of the fundamental formula binding the Talmudic unities of length and capacity, once we are certain that these values E = 2.1825 cm and Log = 547

[9] The formula of Rav Hisda, Pesahim 109a.
[10] See BDD 21 pp. 41 – 42. Indeed $91.1667 * (\pi / 6) = 47.7348$ cm^3.

cm^3 are the correct ones, we must ask ourselves, if this formula 1.5 eggs = 1 Revi'it = 10.8 E^3 is not

- an approximation
- or if this is an ancient formula, which was not adapted when the Rabbis slightly adapted the ancient Rabbinical standard of measure to the Roman standard of measure.
- Or if perhaps the fundamental formula binding the Talmudic unities of length and capacities of Rav Hisda refers to another standard and does not fit our propounded standard of large units of capacity based on Log = 0.547 liters and of units of length based on Etsba = 2.1825 cm.

In the present paper we will adopt a completely new position. Instead of considering that there was one unique Talmudic metric system, we will show that in fact there were apparently three different standards of Talmudic units of measure of volume and capacity, but only one undisputed principal standard of measure of length. These three standards of units of volume are described and mentioned in the Talmudic litterature and they even find their roots in the Mishnah or in the Tossefta. That means that these divergences of appreciation leading to three different metric system, already coexisted by about the year 200 CE the time of the completion of the Mishnah and the Tossefta.
The study of these three standards will allow us to better understand the subject and reconstruct the Talmudic reality.
Two of these standards would later completely disappear and completely be forgotten and only the third standard, the one, which Rav Hisda described, would survive.
This is in contradiction with the generally accepted opinion that there was only one standard of unity of measure of volume and capacity, the standard described by Rav Hisda in B. Pesahim 109a, whose details, except the binding formula, were forgotten with the time, as in fact so many other biblical and rabbinical laws and traditions. The forgetting of the elements of this ancient standard of units of measure of volume transformed the formula of Rav Hisda from the definition of one standard of units of capacity in a general formula allowing the definition of different standards of measure and it brought about the two current main divergent standards of units of volumes, the standard of the large units and that of the small units, both fulfulling the fundamental formula of Rav Hisda. Both standards have their champions and each standard has its variants and presents its difficulty and contradiction.
In fact there is not a perfect standard because each of them has its weak and strong sides.
There is a very abundant rabbinical litterature on this subject ranging from the period of the Gaonim until the modern times. Its access is difficult because the units of length, weight and of volume used by them, often bearing the same denomination, are very misleading. They depended on the geographic area and on the epoch considered. The difficulty is still increased and decupled when it concerns coins, which are often used as units of weight and therefore also as units of volumes, calculated as the weight of water contained. The same denomination generally has a different quantification at different periods and in different areas, mainly because of the debasement of the coinage.

2. The standard of the Talmudical large units of measure of volume and capacity.

This standard was mentioned in Mishnah Terumot X: 8 and developed in Yerushalmi Terumot X: 5.[11] It concerns the quantity of unclean[12] fish that forbids a pickle of fish. The text of the Mishnah is as follows:

משנה:
דג טמא שכבשו עם דג טהור, כל גרב שהוא מחזיק סאתים, אם יש בו משקל עשרה זוז ביהודה שהן חמישה סלעין בגליל, דג טמא , צירו אסור...
גמרא:
כל גרב שמחזיק סאתים:כמה סאתה עבדא? עשרים וארבע לוגין וכמה לוגא עביד? תרתין ליטרין וכמה ליטרא עבדא? מאה זוזים, נמצא כל זין וזין אחת מתשע מאות ושישים...

It concerns the weight of non kosher fish which makes unfit a pickle of two Sa'ah of kosher fish. As soon as the quantity of non kosher fish reaches the weight of 10 Zuz, the whole pickle is unfit.
The Zuz is a denomination equivalent to the dinar. The Dinar corresponds to the Roman Denarius. It is unit of weight of 3.411 gr and the Sa'ah is a volume of 24 Log. The Zin is a denomination equivalent to 10 Zuz. The gemara tells us that each Log weighs 2 Litra and each Litra weighs 100 Zuz. Thus the pickle has a volume of 48 Log and each Log of pure water weighs 200 Zuz. Finally neglecting the difference of density of pure water and the pickle of fish, the weight of this pickle is 2 * 24 * 200 = 9600 Zuz.
Therefore the proportion of non kosher fish making the whole mixture unfit for consumption is 10 / 9600 or 1 / 960.
According to the Talmud of Jerusalem, 1 Log of water weighs 200 Zuz. If we consider, according to the data of the Roman units of weigh that 1 Zuz weighs 3.411 gr, then the weight of 1 Log is 682.2 gr and the Log has a capacity of 682.2 cm^3.
Now if the Zuz represents the coin of 1 Dinar, the fourth part of the Shekel according to the Tyrian standard, then its weigth is sligtly greater and weighs about 3.50 gr and the Log water weighs 700 gr and has a capacity of 700 cm^3. Both these values, 682.2 cm^3 or 700 cm^3 are huge, much greater than any value proposed and greater than the volume of 600 cm^3 adopted by Hazon Ish.
As already mentioned the comparison of the standard of the Talmudic units of volume with that of the Roman units of volumes lets a doubt remain in the Roman attribution of the Log: sextarius 0.547 liters[13] or hemina 0.274 liters. The volume ascribed to the Log is thus exagerated and we must conclude that the Talmud Yerushalmi was dealing with the Sa'ah and the Log of Jerusalem. Therefore the Log midbari weighs only (4/5) 200 = 160 dinars.
The weight of 1 Log midbari of water is then 3.411 gr * 160 = 545.76 gr which corresponds with a good precision to the sextarius[14] of 0.547 l.

[11] According to the numerotation of the Talmud Yerushalmi edited in Vilnius.
[12] Non kosher fish.
[13] Boeckl (1838) established the capacity of the sextarius to 545.75 but the most recent value adopted for the sextarius is 547 cm^3.
[14] We should not be struck with amazement by this result. Indeed first, we have seen that there is a margin of error and uncertainty about the capacity of the Sextarius, second, there is a margin of error in the calculation made in Yerushalmi Terumot. Indeed there is an approximation in this calculation because the

We can conclude that the Mishna Terumot and the Talmud Yerushalmi work according to the standard of the large units of measures of volume. Let us now examine the formulas binding the units of length and capacity:
First formula:
1 Mikveh = 40 Sa'ah = 960 Log = 960 * 0.54576 = 523.93 l = k_1 * 143.713 l.[15]
We deduce : k_1 = 3.6457 instead of 3 in the classical formula :
1 mikveh = 40 Sa'ah = 3 $(Cubit)^3$.[16]
The formula is then : **1 mikveh = 40 Sa'ah = 3.6457 $(Cubit)^3$. (1)**
Second formula:
1Revi'it = k_2 $(Etsba)^3$. Thus 136.44 = k_2 $(2.1825)^3$ = k_2 10.3959.
We deduce : k_2 = 13.1244 instead of 10.8 in the classical formula:
1 revi'it = 10.8 $(Etsba)^3$. [17]
The formula is then : **1 Revi'it = 13.1244 $(Etsba)^3$. (2)**
Note that formulas (1) and (2) are equivalent. If we multiply (2) by 3840 we get (1).[18]

Let us now consider the theoretical minimal Mikveh of one cubit by one cubit section. It contains 523.93 l. and the height of the water is 3.6457 C= 3.6457 * 0.5238 = 1.90 m. If an average man of 75 kg submerges, the apparent volume of water is 523.93 + 75 = 598.93 l and the apparent height of the water is 598.93 l / $(5.238)^2$ dm^2= 21.82 dm = 2.18m. Thus any man can submerge vertically.

Strong sides of this standard of the large Talmudic units of volume.

- Different passages in the Talmud show that the standard of the Talmudic units of measure of volumes and capacity was bound to the Roman standard. Nevertheless both standards are not parallel so that a doubt remains whether the Log must be equalized to the Sextarius (or Kestes in Greek denomination) or Hemina (Kyathos in Greek denomination).
- There are some Talmudic passages, which support that the Log was in fact equal to the Greek denomination Kestes and corresponded to the larger denomination and that the Revi'it corresponded to the Quartarius, the fourth part of the sextarius.

Weak sides of this standard of the large Talmudic units of volume.

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density of the pickle was taken as 1. It is possible that the exact calculation of the Yerushalmi should have been the following. The equation : 1 Litra = 100 Zuz is an approximation. In the Roman metrology 1 Libra = 1 Talmudical Litra = 96 Denarius and 1 Mina = 100 Denarius and by approximation and confusion we find 1 Litra = 100 Denarius. If so the Log of Jerusalem of pure water would weigh 960 * 0.96 = 921.6 Zuz but the pickle with a density of about 1.0417 would finally weigh 960 Zuz. If so the weight of 1 Log midbari of water would be 0.8 * 192 = 153.6 Zuz weighing 153.6 * 3.411 = 523.93 gr if 1 Zuz = 3.411 gr (theoretical weight according to Roman metrology) or 153.6 * 3.50 gr = 537.60 gr if 1 Zuz = 3.50 gr (weight of the coin 'Dinar') according to the tyrian standard. As we see there will always remain a possible margin of error of 4%.

[15] According to the standard of Talmudic units of length that we adopted: the Cubit (Amah) is C = 52.38 cm and C^3 = 143.713 dm^3.

[16] Eruvin 4b and 14b, Pesahim 109b, Yoma 31a and Hagiga 11a.

[17] The formula of Rav Hisda Pesahim 109a.

[18] (1 / 3840) *3.6457 **(**Cubit)^3 = (1/3840) *3.6457 * 13824 **(**Etsba)^3 = 13.1244 **(**Etsba)^3

- According to the former conclusions 1 Revi'it is 136.44 cm^3 and Beitsah, the egg is then 90.96 cm^3. This is not the volume of an average egg of hen. On the contrary, this corresponds to an extra large egg.
- This is a serious problem. The solution, which was proposed, consists in measuring a volume through the number of eggs, which can be stored in this volume.[19]
- Nevertheless the Talmud mentions, even if briefly and nearly on an allusive way, the measure of the volume of eggs by measuring the volume of displaced water.[20] But this is not a refutation of this proposition. According to the champion of the large units of capacity, a volume expressed in a number of eggs represents the number of eggs stored in this volume. This seems the only likely explanation. Now the volume of the parallelepiped circumscribed to the egg, which represents its litter is $6 / \pi$ times the volume of the egg or about twice the volume of the egg. Therefore the measure of the volumes in eggs divided by 2 gives the number of eggs stored in the volume. Thus whatever the definition of the measure of a volume in eggs may be, we must always use the traditional method of measure of the volume of the eggs.
- The standard of the large units of volumes raises difficulties in the explanation of the Sea of King Solomon, "the Yam shel Shelomo" and its contents of 2000 Bat. (1 Kings 1: 23).

Finally we must note that this standard of large units of capacity is undeniably described in the Mishna and Talmud Yerushalmi and constitute a coherent standard of units of capacity. It was nevertheless not correctly understood and was generally ignored.[21]

3. The standard of the Talmudical small units of measure of volume and capacity.

According to the principle that the standard of the Talmudic units of volume and capacity cooresponds to the Roman standard, there is a second possibility to examine, that the Log is equal to Hemina and represents a volume of 274 cm^3 and therefore
40 Sa'ah = 960 Log = 960 * 0. 274 = 263.04 l. Therefore
1 Mikveh = 960 Log = 263.04 dm^3 = 1.83 * (143.713) = 1.83 $(Cubit)^3$.
1 Revi'it = 68.50 dm^3 = 6.5891 * 10.3959 = 6.5891 $(Etsba)^3$.

Such a theoretical standard of Talmudic unites of measure of volumes does not practically exist, but it seems very similar to a standard of unites of measure of capacity mentioned by Rabbi Youssa in Talmud Yerushalmi[22] according which

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[19] See BDD 21 pp. 41 – 42. This solution was proposed by Y. Borenstein.
[20] See Mishna Kelim 17: 6 and Tossefta Nazir, 4: 1.
[21] Rabbi Yom Tov Lipman Heller noted in Ma'adanei Yom Tov, Berakhot III: 30; 80 that the volumes of Y. Terumot X : 5 are three times the small volumes of Maimonides or more precisely 100/35 = 2.8571. In fact because many approximations and imprecision , the true ration was in fact 136.4375 /74.375 = 1.8345. The difference between 3 and 1.83 is huge. Because of the impression of exaggeration, this passage in Y. terumot was not generally taken seriously. It was considered as an individual position and was not followed by the Rabbis and rulers.
[22] Y Pesahim X: 1 (near the end), Y. Shekalim III: 2 and Y. Shabbat VIII: 1.

1 Revi'it = 2E * 2E * 1.8333 E = 7.333 (Etsba)^3. (3)

Tossafot on B. Pesahim 109a could not imagine that this relation describes another standard of unites of measure of volumes and wanted to explain it by the use of fictitious units of length of Tsipori[23] in order to identify it with the formula of Rav Hisda.
If we mutiply equation (3) by 3840 we get the relation corresponding to (1) :
1 Mikveh = 40 Sa'ah = 960 Log = 3840 Revi'it =28159.872 (Etsba)^3 =
(28159.872 / 13824) (Cubit)^3 = 2.037 (Cubit)^3
Hence **1 mikveh = 40 Sa'ah = 2.037 (Cubit)^3. (4)**
We have thus 1 Revi'it = 7.3333 * (2.1825)^3 = 7.3333 * 10.3959 = 76.2364 cm^3
1 Log = 304.9452 cm^3 and 1 Mikveh = 960 Log = 292.747 liters, about 11% above Hemina. The standard defined by Rabbi Youssa is thus very near to the standard of the small unites of measure of volume.

Remark.

If we consider the following variant of formula (3) :
1 Revi'it = 2e * 2e * 1.8 e = 7.2 (Etsba)^3. (5)
and we multiply both members by 3840, we find :
1 Mikveh = 40 Sa'ah = 960 Log = 3840 Revi'it =27648 (Etsba)^3 =
(27648 / 13824) (cubit)^3 = 2 (Cubit)^3
Hence: **1 mikveh = 40 Sa'ah = 2 (Cubit)^3. (6)**
Because of this result we can seriously ask ourselves if Rabbi Youssa had not in mind the formula (6) when he wrote formula (3).
Furthermore, this result confirms us that Rabbi Youssa used our standard of units of length or at least a standard very near to it. Indeed, only the use of that standard of units of measure of length allows this standard to get a Log nearing to the volume of the Roman unit Hemina and the formula (6).

Strong sides of this standard of the small Talmudic units of volume.

- In the present standard 1 Revi'it = 76.2363 cm^3 and 1 egg = 50.8242 cm^3 exactly the volume of an average hen egg.
- Some quotations of Josephus and some translations in the Septuaginta follow the small unites of measure of volume.
- This standard of small units of volume allows explaining the data about the Yam shel Shelomo and its contents of 2000 Bat = 150 Mikveh.
- Many Rishonim adopted small units of measure of volumes. Among them we can note that Rashi,[24] Rabbeinu Gershom[25] and Rabbeinu Hananel[26] write that 1 Log = 1 Litra,[27] in contradiction with Mishna Terumot X: 8.

[23] See BDD 21, p. 32 note 68.
[24] Eruvin 29a and Hulin 110a: ¼ Kav = 1 Log = 1 Litra.
[25] Hulin 110a: ¼ Kav = 1 Log = 1 Litra. Baba Batra 89b : ¼ Kav = 1 Log = 1 Litra.
[26] Eruvin 82b: 1 Kav = 24 Ritel = 24 Litra. Hence 1 Litra = 1 Ritel = 1 Log

- Some references in the translation of the Bible by Jerome and the Septuaginta follow the smaller denomination.
- The archeological discovery of a pot with a capacity of about 22 liters bearing the inscription Bat seems to be an indisputable proof that the standard of small units of capacity was not only a theoretical system but that it really existed despite the different Talmudic quotations mentioned above, which gives the impression that the Talmud favored the standard of the great units of capacity. But from the other side, it must not be exaggerated: it proves that the standard of the small units of capacity was in use in some areas in the time of the Mishna but it does not prove that it was the only standard in use and that the other standards mentioned in the Talmudic texts had no practical existence.
- Asher Grossberg, the renowned reseacher of the old miqva'ot of the Mishna period, focused my attention on the Mikveh of Massada, which had a working volume of 420 l. probably correponding to a theoretical volume of about 332 l. or even less. This volume is much less than the theoretical volume of 40 Sa'ah = 524 l. according to the first standard (large units of volume) and even less than 431 l. according to the third standard, the present halakhic standard used with Etsba = 2.1825 cm.
  Although the people of Massada were behaving according to the highest standards of purity, the volume of their Mikveh was not in agreement with our present halakhic requirements and standard.

Weak sides of this standard of the small Talmudic units of volume.

- We have examined several Talmudic passages, which support that the Log was in fact equal to the Greek Kestes and corresponded to the larger denomination.
- The theoretical volume of the Mikveh is 291.22 l. or 2.037 $(Cubit)^3$. This volume has a section of 1 cubit * 1 cubit and a height of 2.037 cubit. If a man of 75 kg submerges, the apparent volume of water is 291.22 + 75 = 366.22 l. and the apparent height of the water is 2.55 cubit = 2.55 * 52.38 = 133.57 cm. It appears that it is impossible to submerge in a standing position. Note that although the situation looks strange, this is not a reason to disqualify this Miqveh.[28]

<u>The standard of small Talmudic units of volume has roots in the Tossefta.</u>

The Tossefta on Massekhet Kelim, Baba metsia, V: 1 is quite sybiline and the commentators are very reserved and not prolix in its elucidation.

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[27] However the denomination Litra has not the same meaning for the two first Rabbis as for the third. The two first consider probably a litra of 96 * 3.50 = 336 gr. Indeed Rashi didn't probably know the Roman denarius of 3.41 gr but he had probably at his disposal some Sela coins. R. Hananel considered probably a litra of 100 * 4.25 = 425 gr.

[28] Shulhan Arukh Yoreh Deah 198: 35 and 36. Some Mikva'ot examined by Asher Grosberg must be used in lying position.

In the first part of this Braïta, the stamist mentions the minimum volume of the Mikveh: 1 cubit (length) * 1 cubit (breadth)* 3 cubits (height). This seems to be the first historical mention of this dimension.
In the second part of the Braïta, Rabbi Yossi[29] mentions the case of the Sea of King Solomon and explains that the Sea contained 2000 Bat liquid but it was possible to store 3000 Bat by taking into account the volume of solid stored in the tank of the Sea and above the building of the Sea.
The most likely explanation is the following: According to Tana Kamma, who considers that 1 Mikveh = 960 Log = 3 $(\text{Cubits})^3$ =13.3333 Bat,[30] it is impossible to store 2000 Bat of liquid in the Sea. Thus the introduction of the Sea in the debate is a manner to contradict the volume of the Mikveh of Tana Kamma. By contrast, according to Rabbi Yossi, who champions the equation 1 Mikveh = 13.333 Bat ~ 2 $(\text{Cubits})^3$ this is in fact possible.

Let us examine the problem in detail. The Sea of Solomon is a cylinder of external diameter of 10 cubits, internal diameter of 10 Cubits – 2 Handbreadths = 9.6667 Cubits and of height of 5 Cubits.[31] The inner volume is $V = (3.1416 / 4) * (9.6667)^2 * 5 = 366.958\ (\text{Cubits})^3$.
The volume of the Mikveh of hakhamim is 3 $(\text{Cubits})^3$, while that of Rabbi Yossi is probably similar to that of Rabbi Youssa[32] and is about 2 $(\text{Cubits})^3$.
Thus according to Hakhamim the Sea holds 122.32 Mikveh = 1630 Bat and according to Rabbi Yossi and Youssa, it contains 183 Mikveh = 2446.33 Bat.
According to the former calculation we see that Rabbi Youssa had enough reserve to take into account the thickness of the bottom of the tank and even to increase the thickness of the external wall. By contrast the situation is problematic according to Hakhamim and Rav Hisda and therefore in B. Eruvin 14b Rami bar Ezekiel was obliged to propose a far-fetched solution in order to reach the required quantity stored in the Sea.

4. The third standard of Talmudical unites of measure of volume and capacity.

This is the standard generally considered in the Babylonian Talmud and by all the Rabbis of the History, from the Tamudical times until today. The fundamental formulas binding the units of length and volume are:

1 mikveh = 40 Sa'ah = 3 $(\text{Cubit})^3$. **(7)**

and **1 Revi'it = 10.8 $(\text{Etsba})^3$.** **(8)**

The basis of this standard is much more difficult to find and explain. It is clearly a standard intermediate between the two former standards but the principle underlying it, is not clear.

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[29] Likely Rabbi Yossi ben Halafta, one of the important teachers of Rabbi Yehuda ha-Nassi. He belonged mainly to the second century.
[30] 1 Bat = 72 Log, 1 Mikveh = 960 Log = 13.333 Bat.
[31] Those are the dimensions generally accepted by the authors in their calculations.
[32] Rabbi of the fourth century, probably the main craftsman of the modern Jewish calendar. He was the author of the formula (3).

One point is certain, this standard is ancient and is not the result of a mistake. It appears already in the Tossefta[33] and it was existing at the time of the Mishna.
When we consider that the volume of the Log is about 0.546 l. in the first standard and about 0.305 l. in the second standard, the volume of 0.449 l. for the Log according to the third standard, (when we consider Etsba = 2.1825 cm), appears to be near to the mean of the first two values. Maybe that this was the raison d'être of this third standard because of the hesitancy between the two first standards. If this is the explanation of this third standard, then again it would be an indirect justification of the propound standard of the Talmudic units of length that we champion.

5. Historical record of the rabbinical standards of measure.

More than in any other area of Halakha, the forgetting of the preceding standards was the consequence of so many factors, debasement, political changes, emigrations, modifications of the standards imposed by the authorities, even without a political revolution and à fortiori change of regime. The effects of this forgetting were felt in the different areas of the standards of measure.
For example the Rabbis of Babylonia from the time of the Saborayim and Gaonim onwards forgot the knowledge of the units of weight and coinage and assimilated the Dinar (coin and unit of weight) to the weight of an Arabic golden coin of 4.25 gr. instead of 3.41 gr.(weight according to Roman standard of weight) or 3.50 – 3.54 gr (weight of a Dinar according to Tyrian standard) and fixed the weight of the Shekel to 17gr. instead of 14.17 gr (according to the Tyrian standard). Similarly they forgot the length of the Talmudic units of length, the Cubit or Ama, the Handbreadth or Tefah and the thumb or Etsba and also the units of capacity, the Log and the Revi'it and two of the three standards of Talmudic units of measure of capacity.
The only element which was not forgotten was the formulas of Rav Hisda (7) and (8), by contrast to the two other standards of Talmudic units of measure of capacity. But while the third standard of units of measure of capacity was at the origin a well precise standard, it became now a plain formula binding the units of length and capacity, in which it was possible to incapsulate a standard of large units of capacity as well as a standard of small units of capacity.
For lack of more precise data, the rabbis must fall back on the natural values of two basic denominations, the Etsba and the Beitsah, the thumb and the egg, in order to behold likely sizes for the different dimensions.
Champions of large units of capacities as well as champions of small units of capacities were now obliged to work with the same formulas (7) and (8) and therefore the choice of large or small units of capacities, necessarily imposed large or small units of length. Similarly the choice of the units of weight had a direct influence on the units of capacity because the volume of the Log was expressed as the weight of its contents in water and hence on the units of length and volume. So, it appears, the whole Talmudic metric system was in disarray. Nevertheless the rabbis succeeded by the use of natural values to define reasonable, but sometimes exagerated values for the different sizes. Of course, the champions of the large units of capacity and those of the small units, who had before, in the time of the Mishna, their own standard of units of capacity had now to make their

[33] Tossefta on Massekhet Kelim, Baba metsia, V: 1

way through the same and unique formulas (7) and (8) with unpleasant consequences on the units of length.
The data of the metric system of Maimonides, based on the principle of small units of length and capacity were included in his Hibbur and in Shulhan Arukh and so entered Halakha. The formula (7) was introduced by Maimonides in Hilkhot Kelim 3: 4
And in Hilkhot Miqva'ot 4: 1. He ruled[34] that the Rev'it has a capacity of 17.5 * 4.25 = 74.375 gr. of water, hence a capacity of 74.375 cm^3.
He ruled[35] also that the Litra of water weighs 35 dinar, the rev'it 17.5 dinar and the Log 70 dinar. Thus the volume of the Litra is 148.75 cm^3, that of the Log is 297.5 cm^3 and that of the Revi'it is 74.375 is cm^3. We note that the Revi'it of Maimonides is very near to the Revi'it in the ancient standard of the Talmudical small units of measure of volume and capacity of 76.2363 cm^3.[36]

In fact the Shulhan Arukh ruled according to Maimonides, without nevertheless setling the dispute, as the champions of the large units of capacity did not relent.
In Shulkhan Arukh, Yoreh Deah 294:6 It writes : 1 Ma'ah = 16 Barleycorn= 0.25 Dirham.
We know that in Maimonides' Hibbur: 1 Dinar = 96 barleycorn = 4.25 gr. and 1 Dirham = 64 barleycorn, hence 1 Dirham = (2/3) * 4.25 = (1/6) Sela = 2.8333 gr.
In Shulhan Arukh, Yoreh Deah 305:1 it writes 1 Sela = 120 Ma'ah = 30 Dirham
Hence 1 Sela = 17 gr.
In Shulhan Arukh, Yoreh Deah 324:1 it writes thagt the volume of flour imposing the deduction of Hallah, which is a volume of flour of 43.2 eggs, which is equal to 1 Issaron = 7.2 Log = 28.8 Revi'it, weighs, in the case of Egyptian flour, 520 Dirham. This wheight corresponds to 86 2/3 Sela = 1473.33 gr. Rema adds that it corresponds to a volume of <u>about</u> 311, 1111 $(Etsba)^3$. All these data were already given in Maimonides' Hikhot Bikurim 6: 15.[37]
From these elements we can conclude that Shulhan Arukh fully endorses the complete metric system of Maimonides, weight, coinage and capacity.
But as already mentioned, this did not settle the dispute. Even those who accepted Maimonides' conclusions must surmount the difficulty translating Maimonides' data in local units of measure.[38] But after that the contradiction between the natural measures of thumb and eggs resulting from the fundamental formula **1 Revi'it = 10.8 $(Etsba)^3$** were revealed, the champions of the great units of measure of capacity appeared. In fact this was long process. In the very beginning of the 14th century Rabbi Solomon ben

[34] Hilkhot Eruvin 1: 12.
[35] Hilkhot Eruvin 1: 12.
[36] But this is rather a pure chance because the two standards rest on completely different assumptions.
[37] In fact 1 Revi'it = 10.8 $(Etsba)^3$ and therefore 28.8 Revi'it = 311.04 $(Etsba)^3$. The data given by Maimonides correspond to a quantity of Egyptian flour of 28.8 Rev'it volume weighing 520 dirham, hence a weight of 520/28.8 = 18.0556 Dinar/Revi'it. Furthermore the Revi'it of water weighs 26.25 Dirham (Hilkhot Eruvin 1:12). Therefore the density of Maimonides' flour was 18.0556/ 26.25 = 0.6878 kg/l. Today we get a density of 0.58 – 0.6. Therefore the weight of flour requiring the deduction of Hallah is 74.375 * 28.8 = 2142 cm^3 and it weights 2142 * 0.59 =1263.78 gr. rounded off to 1250 gr. The same calculation according to Hazon Ish gives: Volume : 43.2 * 100 = 4320 weighing 4320* 0.59 = 2548 gr, In fact they prononce the benediction from 2250 gr. onwards.
[38] Apparently this seems the main reason of the work of R. Hayyim Naé.

Menahem Meiri of Perpignan, noted in *Beit ha-Behira*,[39] that the measure of volumes by the system based on Etsba is safer than by the eggs.
During the 15th century R. Simeon ben Tsemah Duran noted for the first time that the Mikveh determined by the volume of three cubic Cubits, measured from the natural breadth of thumb, leads to much bigger eggs than the normal average-sized eggs.
The problem took its full extent in Europe[40] among the Ashkenazi Rabbis, at the end of the 18th century when R. Ezekiel Landau from Prague noted that the volume of pastry, which obliges to Hallah, measured by the volume of 43.2 eggs, is half the volume of 28.8 Revi'it measured by the rule: 1 Revi'it = 10.8 $(Etsba)^3$**.** This was the starting point of the new school of the champions of the large units of capacity.
It is interesting to note that no one, never, questioned the validity of the formula of Rav Hisda and realized that this formula is valid only for one group of ''correct'' values of E, Etsba=2.1825cm and Revi'it=112.2759 cm^3 representing a standard intermediate between the large and the small unites of volume.

6. Maimonides' metric system, emulating the Talmudic Standard II.

The metric system described in his Hibbur represents Maimonides' final position on the subject. It differs slightly from his position in his commentary on the Mishna.[41] The basis assumptions are the following: Maimonides follows the metric system of the Gaonim and the measure made by Rabbi Hilai Gaon. He identifies the weight of the Talmudic siver Dinar with the weight of the arabic Gold Dinar weighing 4.25 gr. He writes the the weight of 1 Revi'it water is 17.5 Dinar or 17.5 * 1.5 = 26.25 Dirham because 1 Dinar = 1.5 Dirham and 1 Dirham = 2.8333 gr. This result is equivalent to the measuring made by rav Hilai Gaon of Sura, according which the weight of 1 average egg is 16.6666 Babylonian Zuz.
The Babylonian Zuz is connected tot the Arabic Dinar by the following relation:
10 Babylonian Zuz = 7 Arabic Dinar. This relation is quoted in the commentary of Rashi on Gemara Bekhorot 50a related to the text of the Gaonic interpolation:

והנך לפדיון הבן [דהוי להו עשרין מתקלי במתקלי דינרא דאינון עשרין ותמניא זוזי ופלגא ופלגא דדנקא]

The 20 Tyrian Dinars of Pidion ha-ben are equivalent to (28 + 0.5 + 1/12) = 28.5833 Babylonian Zuz = Babylonian Dirhams. More precisely 20 Tyrian Dinars = 20 /0.7 = 28.5714 = (28 + 0.5 + 1/14) Babylonian Dirhams.
Rashi explained the relation between the Tyrian dinar and the Babylonian Dirham in Bekhorot 50a at the 24th line:

דכיוון דשבע צורים עושין עשרה בבליים...

7 Dinars of the Tyrian Standard have the same weight as 10 Byabylonian Dirhams.
Thus 1 Babylonian Zuz weighs 0.7 Dinar = 0.7 * 4.25 = 2.975 gr.
The average egg of Rav Hilai Gaon weighs 16.6666 * 2.975 = 49.5833 gr and his Revi'it weighs 49.5833 * 1.5 = 74.375 gr = 17.5 Dinar * 4.25 gr.

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[39] Eruvin 83b and Pesahim 109a.
[40] The problem was still raised before in different instances. See Benish (1987) pp. 63 – 68 and Weiss (1984) p. 372. See also the introduction to Mikraot Gedolot, Venice 1648.
[41] See BDD 21 p. 45.

The weight of the egg of Maimonides is thus exactly that of Rav Hilai Gaon[42] and his metric system appears to coincide with that of the Gaonim. From the Revi'it, we find Etsba = 1.9025 cm by the fundamental relation (8): .

Thus apparently Rambam accepted the volume of an average egg, as measured by Rabbi Hilai Gaon as depature point and calculated Etsba through the formula of Rav Hisda and accepted that the value of Etsba is a conventional unit of measure, slightly smaller than the natural value.

We deduce also the following formulas: 1 Revi'it = 17.5 Dinar. (9)

1 Litra = 2 Revi'it = 35 Dinar. (10)

1 Log = 4 Revi'it = 70 Dinar. (11)

The formula 1 Litra = ½ Log recalls a similar frormula in Mishnah Terumot X: 8 but the similarity stops here. In fact Rambam called the half Log Litra, but the three formulas are experimental and are base on the weighing of Rabbi Hilai Gaon. The formula (10) is nevertheless surprising as we are accustomed to 1 Litra = 96 dinar or 100 dinar.

In fact Maimonides changed the definition of the Litra with regard to the traditional denomination Litra in the Talmudic metric system and the denomination Libra in the Roman metric system.

As mentioned above, Rambam adopted the weight of the Dinar fixed by the Gaonim, a Dinar of 4.25 gr. And a Shekel or Sela of 17gr.

Now we know that the extant coins of Shekel weigh from 13 to 14 gr and their theoretical weight is considered to be 14.16 gr according to the Tyrian standard.

Furthermore we know that Rambam knew these coins and had probably some exemplars at his disposal as we learn from a quotation from one of his responsa.[43] The explanation seems to be the following: Rambam considered that the coins at his disposal with the ancient Jewish writing were Shekalim from before the re-evaluation of 20% discussed in Bekhorot 50a. Therefore the Talmudic Shekel must be fixed at 14.16 * 1.2 = 17 gr.

Remark.

In his commentary on the Mishna, Rambam has a slightly different metric system.

From his commentary on Mishna Bekhorot 8: 8, we learn that 1 Dirham = 61 Barleycorn

From his commentary on Eduyot 1: 2 we learn that 1 Revi'it water weighs 27 Dirham.

We have then: 1 Dirham = (61/96) * 4.25 = 2.70 gr and 1 Revi'it water = 27 * 2.7 = 72.9 gr instead of 17.5 * 4.25 = 74.375 gr.

This change in Maimonides' position about the weight of the Revi'it of water can be understood. Despite his own measure, as he underscored, Rambam preferred to accept in his Hibbur the value of the Gaonim. But other changes raise great interrogations and difficulties because a change in the value of the Dirham during his lifetime should have been underscored by him. Furthermore we don't understand why the weight of one Omer of Egyptian flour remained unchanged at 520 Dirham while the Dirham was re-evaluated by 4.93% ?[44]

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[42] References to the weighing of Rav Hilai Gaon can be found in Benish: Midot ve shiurei Torah, Benei Berak 1987, pp. 216 – 217.

[43] Responsum n° 468 in Tshuvot ha-Rambam, Joshua Blau, Jerusalem 1960, vol 2, p, 513

[44] These problems were already raised in Jacob Weiss: Midot U Mishkalot shel Tora, Jerusalem 1984, p. 159.

The relation, today universally accepted and ruled in Shulhan Arukh,[45] that 1 Dinar = 1.5 Dirham is derived from Hilkhot Bikkurim 6: 15 where he writes that 520 Dirham = 86 2/3 Sela = 346. 66667 Dinar. I have proposed to solve the conundrum by invoking a lapsus calami in the figure 86 2/3 and correcting it to 82 2/3. But I agree that this correction is questionable… all the more that there is no manuscript evidence and it would then be the author's mistake! But it would solve the contradiction of the re-evaluation of the Dirham.

7. Rashi's metric system, emulating the Talmudic Standard II.

Rashi and Rabbeinu Gershom wrote : 1 Log = 1 Litra. There remain a margin of incertitude about the weight of the Dinar of Rashi and about the number of Dinar in his Litra. The weight of the Sela at Rashi and Rabbeinu Gershom's disposal was about 14 gr or slightly less according to the condition of the coins at his disposal.[46]
Rashi and Rabbenu Gershom made a difference between Mana and Litra. This was also the case in the German coinage, which was emulating the Roman coinage. We can find a proof of this statement in the following quotation from Rabbenu Gershom in Bekhorot 49b and in Hulin 84a:

דנר זהב הוי ה' פשוט למשקל הברזל, דינר כסף ב' ומחצה למשקל הברזל. בכורות מ"ט ע"ב.

והמנה הוא ליטרא וחצי אונקיה. חולין פ"ד ע"א.

The Pashut refers to the Esterling coin of 1.4 gr. Thus according to the first quotation
1 Dinar = 2.5 Esterrling coins = 2.5 * 1.4 = 3.5 gr.
1 Ounce = 20 Esterling coins
1 Mana = 250 Esterling coins = 250 * 1.4 = 350 gr.
1 Litra = 240 Esterling coins = 240 * 1.4 = 336 gr.
Indeed, Rabbenu Gersom wrote in Hulin :
1 Mana = 240 Esterling coins + 10 Esterling coins = 250 * 1.4 = 350 gr.

Thus 1 Log = 1 Litra = 336 gr or 1 Log = 0.336 l.
The formula 1 Log = 1 Litra seems also to be an experimental formula based on the estimation of the volume of average eggs.

8. Rabbi Hayyim Naé's metric system, emulating Maimonides' metric system.

When rabbi Hayyim Naé proposed his metric system emulating Maimonides' metric system and the Talmudic standard of units of volume, the correct quantification of Maimonides' data was not yet completely elucidated. On scientific level, the method of R. Naé is very problematic. His system is based on contradictory and anachronistic assumptions.

[45] Shulhan Arukh Yoreh Deah 294, 6 and 305, 1. See also Kessef Mishneh on Hilkhot Bikkurim 6:15 and on Hilkhot Kelei ha-Mikdash 3: 3.
[46] About the metrology of Rashi, see: Laurence J. Rabinovich, Rashi's Metrology: Money, Coins and Currency from Cologne, Constantinople and the Classical Past, The Manchester 2008 Conference Volume.

He depart from the relation in Maimonides' commentary on Mishna[47]: 1 Revi'it water weights 27 Dirham. Rabbi Hayyim Naé adopts for the Dirham a weight of silver of 3.2 gr. Based on the Turkish Dirham of the nineteenth century and the beginning of the 20th century.[48] Furthermore he deducts the weight of the Dinar by the relation : 1 Dinar = (96 / 64) * 3.2 = 4.8 gr. By so doing he commits not only a dangerous anachronism (confusing Maimonides' Dirham of 2.7 gr in the Mishna, with a modern Turkish Dirham of 3.2 gr) but he forgot that in the commentary of the Mishna the Dirham weighs 61 Barleycorns while in the Hibbur and the Shulhan Arukh it weighs 64 Barleycorns. He made thus an unhappy mixture: 27 Dirham for a Revi'it water is a data in the commentary of the Mishna (instead of 17.5 * (96/64) =26.25 in the Hibbur) and 64 is the weight of the Dirham in the Hibbur (instead of 61 Barleycorns in the commentary of the Mishna). The result of this mixture cannot be very significative.

Thus 1 Dinar = (96 / 64) 3.2 = 4.8 gr and 5 Shekel = 20 Dinar = 96 gr.
1 Rev'it = 27 * 3.2 = 86.4 gr of water, hence a voume of 86.4 cm^3.
And the relation 1 Revi'it = 10.8 $(Etsba)^3$ gives us 1 Etsba = 2 cm.
Now that Maimonides' metric system is well known, we can be surprised that the metric system of Rabbi Hayyim Naé remains reverred and popular. In fact we note that it is similar to the assumed and more coherent position of Rashi.

9. Rabbi Abraham Karelitz' metric system (Hazon Ish), emulating the Talmudic Standard.

Hazon Ish based himself on the reasoning of Rabbi Ezekiel Landau and based all hiy system on the breadth of average thumbs of 2.4 cm and the Cubit of 57.6 cm. This gives us a Revi'it of 149.30 cm3, which he rounded off to 150 cm3. He kept the dinar of Rabbi Hayyim Naé.

[47] Commentary on the following Mishnahs : Peah 8: 5, Shevi'it 1: 2, Hallah 2: 6, Terumot 10: 8, Eduyot 1: 2, Menahot introduction, 5th part, Menahot 9: 2, Bekhorot 8: 8, Kelim 2: 8 and Mikvaot 3: 1.
[48] This weight of 3.2 gr is in good correspondence with the indication given by Rabbi Hayyim Palaggi for the Dirham of his time in his Aggada shel Pessah with the Commentary'' Hayyim Lerosh ''. See Jacob Weiss: Midot U Mishkalot shel Torah, Jerusalem 1984.

Reconstitution of the three original Talmudic standards of units of volume.

	Standard I: Large units of measure	Standard II: Small units of measure	Standard III: Mean Units of measure
Beitsah	90.96 ml	50.824 ml	74.85 ml
Revi'it	136.44 ml	76.236 ml	112.28 ml
Litra	272.88 ml	152.473 ml	224.55 ml
Log	545.76 ml	304.946 ml	449.10 ml
Kav	2.183 l	1.220 l	1.796 l
Hin	6.549 l	3.659 l	5.389 l
Sa'ah	13.098 l	7.812 l	10.778 l
Bat	39.294 l	21.956 l	32.335 l
Omer	3.929 l	2.195 l	3.233 l
Kor = Homer	392.94 l	219.170 l	322.86 l

Tableau 1: Reconstitution of the three Talmudic standards of units of measure of capacity. The assumptions of the different standards are the following: Standard 1, Large units of capacity: Dinar = 3.411 gr Etsba = 2.1825 cm, Cubit = 52.38 cm, Log = 545.76 ml. Standard 2, small units of capacity: Dinar = 3.411 gr Etsba = 2.1825 cm, Cubit = 52.38 cm, Log = 304.95 ml. Standard 3, mean units of capacity: Dinar = 3.411 gr Etsba = 2.1825 cm, Cubit = 52.38 cm, Log = 449.10 ml.

The four modern (post-Talmudic) halakhic standards of units of volume.

	Rambam: Small units of measure.	Rashi, assumed Small units of measure	R. Naé: Small units of measure	Hazon Ish: Large units of measure
Beitsah	49.58 ml	59 ml	57.60 ml	100 ml
Revi'it	74.37 ml	84 ml	86.40 ml	150 ml
Litra	148.75 ml	168 ml	172.80 ml	300 ml
Log	297.5 ml	336 ml	345.60 ml	600 ml
Kav	1.190 l	1.344 l	1.382 l	2.40 l
Hin	3.570 l	4.032 l	4.147 l	7.20 l
Sa'ah	7.140 l	8.064 l	8.294 l	14.40 l
Bat	21.420 l	24.192 l	24.860 l	43.2 l
Omer	2.142 l	2.419 l	2.488 l	4.32 l
Kor = Homer	214.20 l	241.92 l	248.84 l	432 l

Tableau 2: The three present standards of units of measure of capacity. The assumptions of the different standards are the following: 1 Revi'it = 10.8 $(Etsba)^3$. Standard 1, Rambam, small units of capacity: Dinar = 4.25 gr, Etsba = 1.9025 cm, Cubit = 45.66 cm. Standard 2. Rashi, small units of capacity: Dinar = 3.50 gr., Etsba = 2.016 cm, Cubit = 48.39 cm. Standard 3, R. Naé small units of capacity: Etsba = 2.0 cm, Cubit = 48 cm. Standard 4, Hazon Ish, large units of capacity: Etsba = 2.40 cm, Cubit = 57.6 cm.

10. Summary.

1. There was only one standard of Talmudic units of length. The Talmudic Mile was identical to the Roman Mile. It was the diagonal of a square having sides of 2000 Cubits.
   1 Mile = 1481.5 m, Etsba = 2.1825 cm, 1 Cubit = 52.38 cm.
2. Three different standards of Talmudic units of capacity coexisted at the period of the Mishna and the talmud.

- Standard of large units of capacity described in Mishna Terumot.
  Etsba = 2.1825 cm, Revi'it = 136.44 cm^3, Log = 545.76 cm^3.
  Mikveh = 3.6457 $(Cubit)^3$. 1 Revi'it = 13.1244 $(Etsba)^3$.
- Standard of small units of capacity described by Rabbi Youssa.
  Etsba = 2.1825 cm, Revi'it = 76.24 cm^3, Log = 304.95 cm^3.
  Mikveh = 2.037 $(Cubit)^3$. 1 Revi'it = 7.3333 $(Etsba)^3$.
- Standard of medium-sized units of capacity described by Rav Hisda.
  Etsba = 2.1825 cm, Revi'it = 112.25 cm^3, Log = 449 cm^3.
  Mikveh = 3 $(Cubit)^3$. 1 Revi'it = 10.8 $(Etsba)^3$.

3. During the period of the Gaonim the precedent standards were forgotten and only the two formulas Mikveh = 3 $(Cubit)^3$. 1 Revi'it = 10.8 $(Etsba)^3$ were remembered and were considered as authoritative for any metric system. The respect of these formulas put a brake on the undertaking of a rabbinic metric system based on natural values of Etsba and Beitsah and was the cause of the anomaly observed at the end of the 18^{th} century in the metric system: when the Beitsah had a normal volume of an average hen egg, then the Etsba was too small and when the Etsba had a normal value, the volume of the Beitsah was much too important.

4. Maimonides' metric system was certainly similar to the standard of the Gaonic units and it remained authoritative in the Sephardic world until today. It is very similar the Standard II of the small units of measure of Rabbi Youssa with the exception of the value of the unit of length Etsba and the value of the Talmudic Sela.

5. In Germany the Rabbis of the tenth and eleventh century in General and particularly Rashi and Rabbenu Gershom had a very good knowledge of the Roman and Talmudic metric systems. This German metric system was authoritative in Europe until the end of the 18^{th} century at the outbreak of the crisis of the rabbinic standard of units of measure of length and capacity.

6. Today two standards coexist: the standard of the small units of Rabbi Naé which is very similar to the German standard of units of capacity ascribed to Rashi in the present paper and the standard of Hazon Ish which is very similar to the ancient Talmudic standard of the large units of capacity of the Mishna Terumot.