# בדד

בכל דרכיך דעהו

כתב־עת לענייני
תורה ומדע

32
אב תשע"ז

הוצאת אוניברסיטת בר־אילן

CONTENTS

*Book Review*

J. Jean Ajdler[1] & Israel Eichenstein[2]

# Between Ptolemy and al-Battānī: Elements of the Astronomy of Rabbi Abraham bar Hiyya According to his Book *Sefer Heshbon Mehalekhot ha-Kokhavim*

Thanks to several existing manuscripts, one of the books of R. Abraham bar Hiyya, *Sefer Heshbon Mehalekhot ha-Kokhavim*, has survived until today. In 1959, the Spanish scholar J.M. Millás Vallicrosa published the Hebrew text of the first part of the book with a Spanish translation. He also published several tables from the second part of the book. He apparently chose several tables connected to the Jewish calendar that seemed to have an original character. He thus ignored most of the tables, and the few tables that were published are not devoid of mistakes, generally originating from the Berlin manuscript used. In the present study, using MS Malatestiana, MS Paris 1046, and excerpts of MS Berlin as reference manuscripts, we examined the various tables and tried to understand their construction. This enabled us to explain their origin and debug mistakes found in the tables published by Millás Vallicrosa. It appears that the main corpus of the tables rests on the assumptions of Ptolemy. Nevertheless, the radices of the tables (the astronomical parameters at the epoch adopted by R. Abraham bar Hiyya) are deduced from al-Battānī's tables. The author gave no explanation or justification for this procedure. We suggest that, despite the good correlation between the length of Ptolemy's tropical year and the Jewish calendar on the one hand, and the greater confidence in Ptolemy's tables in the short run on the other hand, he was aware of their insufficiency for the long run, notably for a span of time of about a thousand years between the time of Ptolemy's tables (137 CE) and the epoch adopted by Abraham bar Hiyya, on 21 September 1104 at noon. At the end of the book, we find correction tables

1 Civil engineer.

2 אברך כולל 'חלקת יעקב' בבני ברק. Israel Eichenstein provided important manuscripts of the tables of Rabbi Abraham bar Hiyya, also called *Luhot ha-Nassi*, corresponding to the second part of the book *Sefer Heshbon Mehalekhot ha-Kokhavim*. Excerpts of the Berlin manuscript, manuscript No. 649, the manuscript of the Library of Cessena-Biblioteca Malatestiana, Pluteo sinistro XXIX 4 (Malatestiana S XXIX 4), www.Malatestiana.it/manoscritti and MS Paris 1046. It should be noted that this last manuscript, from folio 47a onward, contains material that does not belong to Abraham bar Hiyya, but to ibn Ezra (tables and text).

allowing the finding of al-Battānī astronomical positions through the correction of Ptolemy's position by the addition or the subtraction of a corrective term.

## INTRODUCTION

R. Abraham bar Hiyya was a Jewish scholar born in Barcelona in about 1070. He lived most of his life in Christian northern Spain, during the second part of the 11th century and the first part of the 12th century. It is generally assumed that he died in 1136 in Barcelona, where he had spent a great part of his life. Few details of his life are known. We do know that he had a profound knowledge of Arabic, mathematics, and astronomy, and was renowned as a philosopher, a geometer, a mathematician, and an astronomer.

He was recognized in both the Jewish community and Gentile society. Indeed, he was known by two titles. The first, *Savasorda*, a corruption of the Arabic, *Sahib-al-Shurta*, means "the captain of the bodyguard." This must be a title disconnected from its original meaning,[3] denoting a functionary whose duties were probably within his scope, whether his linguistic abilities, his mathematical and astronomical knowledge, or his skill in surveying (land measuring, equal division in properties). The other title, *nasi*, was probably an honorific title in the Jewish community, its exact significance remaining unknown.

Because of the scope of his competence, Abraham bar Hiyya's specialty was the translation of scientific works written in Arabic. He collaborated in this work of translation with Plato of Tivoli,[4] an Italian mathematician and astrologer who lived in Barcelona for many years and translated scientific works from Arabic to Latin.

Four truly scientific works, two mathematical works, and two astronomic

3 Titles that seem anachronistic exist in all societies; see Foz (1998). We already find a similar situation in Genesis 39: 1. In general, for the biographic elements see *Encyclopedia Judaica:* entry Abraham Bar Hiyya, Baer (1961), Rashed (2003), and Millás Vallicrosa (1949).

4 Plato de Tivoli lived during the first half of the 12th century. He was older than a second, reputed mathematician–translator, Gerard of Cremona (1114–87). He lived in Barcelona between 1132 and 1146, and translated al-Battānī's astronomical treatise into Latin, as "De motu stellarum" ("On the motion of the stars"). The printed edition of this translation appeared in Nuremberg in 1537 with annotations by Regiomontanus, and it was re-edited in Bologna in 1645. A Spanish translation was made in the 13th century, and this and Plato of Tivoli's Latin translation have survived. Delambre knew al-Battānī's book through the 1645 edition.

works, translated into Latin, carry the name of Plato de Tivoli, with or without the name of Abraham bar Hiyya as a translator.

- The *Liber Embadorum* ("Book of Areas" or "Practical Geometry," in Hebrew *Hibbur ha-Meshiha ve ha-Tishboret*) of Savasorda. This book greatly influenced the development of geometry.[5]
- The *Spherica* of Theodosius of Bethynia.[6]
- *al-Battānī's al-Zij* ("Astronomical Treatise").[7]
- *De usu astrolabii* of Abul Qasim Maslama.[8]

A list of Abraham bar Hiyya's works follows:

**Mathematical Works**

- *Yessodei ha-Tevunah u-Migdal ha-Emunah*. M. Steinschneider, Hebraeische Bibliographie, Vol. 7. Spanish translation by J.M. Millás Vallicrosa, 1952.
- *Hibbur ha-Meshiha ve ha-Tishboret*. M. Guttmann (2 parts, 1912–13). Spanish translation by J.M. Millás Vallicrosa, 1931. Latin translation, Plato de Tivoli, 1145.

**Astronomical and Geographical Works**

- *Tsurat ha-Arets ve-Tavnit Kaddurei ha-Rakia*, Basle 1545/6, and Jonathan ben Joseph of Radunia, Offenbach, 1720. Spanish translation by J.M. Millás Vallicrosa, 1956.
- *Heshbon Mehalekhot ha-Kokhavim* edited with a Spanish translation by J.M. Millás Vallicrosa, 1959.[9]
- *Sefer ha-Ibbur*, Filipowski, London, 1851.[10]

5 This translation was completed in 1145. Because of the date, it is unlikely that Abraham bar Hiyya took any part in this translation. However, it does point to his importance, as his Hebrew work was considered worth translating into Latin. Abraham bar Hiyya is credited mainly with the dissemination of the quadratic equation in the Occident through the books of the history of science.

6 Greek geometer of the 11th century.

7 This explains why Abraham bar Hiyya knew al-Battānī's work so well. al-Battānī was an exceptional ninth-century astronomer.

8 Maslama al-Majriti, a Muslim astronomer, mathematician, and scholar in Islamic Spain (Andalusia); died in 1007/8. He improved the existing translation of the Almagest, and introduced and improved the tables of al-Kwarizmi.

9 The Hebrew text of this edition was photocopied in *Poel ha-Shem* without any reference or credit.

10 Completed in 1122. It is generally accepted that Maimonides had this book in mind when

**Philosophical Works**

- *Hegyon ha-Nefesh ha-Atsuvah*, Freimann,[11] Leipzig, 1860. English translation, G. Wigoder, "Meditation of the Sad Soul," 1969.
- *Megillat ha-Megalleh* (also called *Sefer ha-Kitsin* by R. Abraham ibn Ezra),[12] Posnanski, 1924. Spanish translation by J.M. Millás Vallicrosa, 1929. Eschatological book, dealing with the calculation of the era of the redemption and the resurrection.

**The Book *Heshbon Mehalekhot ha-Kokhavim***

We know the book through its edition by J.M. Millás Vallicrosa, which includes the Hebrew text of the first part of the book, its Spanish translation, and the publication of several astronomical tables belonging to the second part of the book.

Taking into account the significant number of extant manuscripts of this book, we must conclude that it was a popular book. In fact, this book does not contain any original elements that would explain this success. It seems to have been popular because it enabled those Jews who did not speak or understand Arabic and Latin to study astronomy, calculate conjunctions, equinoxes, and eclipses, and be on a par in one of the most popular sciences of the day.

The book comprises two parts. The first part is a textbook that explains, describes, and indicates the use of the table. It is thus the canon of the tables. A

he wrote in his commentary on the Mishnah Erakhim 2:2:
**וכבר חיבר זולתינו בספרד בזה הענין וזולתו מהמין הזה חיבור נאה מאוד שאין בינו ובין החיבורים שחברו במזרח בעניני העיבור דומיא בשום צד.**

11 Isaac Eizik Freimann of Cracow, died in 1886.

12 R. Abraham ibn Ezra (1089–1164) quotes this book and contradicts it in his commentary on Daniel 11:30:
**"...וכלל אומר דבריו בדברי קץ, גם דברי רבי שלמה בן גבירול ז"ל רצה לקשור הקץ במחברת הגדולה על שני הכוכבים העליונים. גם דברי רבי אברהם הנשיא בספר קצים ודברי היוצר ודברי רבי יצחק בן לב וכל החושבים המלות או האותיות בחשבון גימטריא, הכל הבל ורעות רוח כי דניאל לא ידע הקץ...".**
R. Abraham bar Hiyya could also have known the younger Abraham ibn Ezra, as we find a table of the solar declination entitled: החכם אבן עזרא. נטית השמש אל (page 11b in the Malatestiana manuscript). The position of the table on p. 11b could indicate that it belongs to the original text and is not an addition. In this table, the maximum declination of the sun is 23; 33. 8°. It corresponds to the last measurements by Arab astronomers and clearly contradicts Ptolemy. It follows a first table of declination constructed on the basis of a maximum declination of 23; 51, 20° according to Ptolemy. In MS Paris 1046, folios 7b-8a, we find the same tables with an inscription at the top of p. 8a: **לאבן העזר נ"ג** ? By contrast, we find at the top of folio 47a: **ר' אברהם אבן עזרא**.

second part consists of a set of astronomical tables.

In order to allow the reader to get acquainted with the subjects treated in the first part of the book, we detail here its table of contents.

### The Book *Heshbon Mehalekhot ha-Kokhavim*

### First Part: Table of Contents

Chapter 1. The measure of arcs of circumference in degrees, minutes, and seconds. Addition of arcs, multiplication, and division of an arc.

Chapter 2. The axis of the celestial sphere and the poles of the equator. Trigonometric lines: sine and cosine.

Chapter 3. Calculation of the declination of the points of the ecliptic. Use of the tables.

Chapter 4. Point of the equator rising together with a point of the ecliptic or the extremities of the signs of the zodiac, at the horizon of a location situated on the equator. In other words, point of the equator rising together with a point of the ecliptic on Sphaera recta (the celestial sphere of those living at the equator with the equator and the parallels perpendicular to the horizon) or simply the right ascension of the points of the ecliptic.

Chapter 5. Oblique ascension of the points of the ecliptic, i.e. point of the equator rising together with a point of the ecliptic at the horizon of a location different from the equator, presenting latitude different from zero.

Chapter 6. Determination of the latitude of a location. Calculation of the latitude of a location from the difference between the length of the day with regard to the average value of 12 hours. Conversely, calculation of the length of the day and the difference with regard to 12 hours from the latitude. Calculation of the declination of the points of the ecliptic. Calculation of the azimuth of the intersection point of the equator with the eastern horizon (the ascendant), and the right ascension of the intersection of the equator with the meridian (the culmination).

Chapter 7. Tangent and cotangent. The tangent of the altitude angle of the sun is equal to the length of the shadow of a horizontal gnomon on the vertical plane perpendicular to it, divided by the length of the gnomon. The cotangent of this angle is equal to the length of the shadow of a vertical gnomon on the ground, divided by the length of the gnomon.[13] Determination of the altitude of the sun through the knowledge of one of these shadows. Calculation of the hour of the day, or the number of remnant hours of the day, from the longitude of the sun. Conversely, calculation of the longitude of the sun from the knowledge of the length of the day.

Chapter 8. The length of the tropical year adopted in this book. Correspondence between the Jewish years of the Era Mundi with the Egyptian, Roman, and Arabic years. Transformation of dates between these different calendars.

Chapter 9. Equation of the days or, in modern language, the equation of time. It is the accumulated difference between the length of the true days with regard to the average length of 12 hours. Transformation of true time into mean time and conversely.

Chapter 10. Mean and true Movement of the sun, the moon, and the five planets. Movement of their ascending and descending nodes. Explanation of the tables.

Chapter 11. Calculation of the latitude of the moon and of the planets. Tables.

Chapter 12. Calculation of the conjunctions and oppositions of the moon. Visibility of the new moon.

Chapter 13. Calculation of the lunar eclipses from the tables constructed for that purpose.

Chapter 14. Parallax of the moon, i.e. the difference between the topocentric and the geocentric moon. In other words, it is the difference between the position of the apparent moon as seen from the surface of the earth and the theoretical position of the moon calculated as if the earth was concentrated at its center.

Chapter 15. Calculation of the solar eclipses from the tables constructed therefor.

Chapter 16. Occultation and reappearance of the planets under the light of the sun.

Chapter 17. Movement of the fixed stars, of the apogees (precession), and ascending nodes of the five planets. Radices of these quantities at the epoch adopted in this book, Wednesday, 29 Elul 4864 or 21 September 1104 at noon in Jerusalem, longitude 67.5° or 22.5° west of the center of the inhabited world (*Tibbur ha-Arets*). Names of the stars of first and second size.

Chapter 18. Astrological chapter. Calculation of the ascendant and the culmination

13 See Sarfati (1968), pp. 109-10.

of the zodiacal signs, i.e. the points of the equator rising at the horizon together with the extremities of the zodiacal signs and crossing the meridian (superior transient of the meridian). Calculation of the twelve astrological houses.

Chapter 19. Calculation of the date in the Egyptian calendar of the recurrence of the sun to a given longitude or to a chosen conjunction.

The text indicates that the excess of a tropical year with regard to an Egyptian year is 88;48°. This must be understood as follows: one day is 360° and therefore 88;48° represents 0.24666 day or 5h 55m 12s. The tropical year is thus 365d 5h 55m 12s: It is exactly the year of Ptolemy.

Chapter 20. Planar representation of the celestial vault of heaven, and with stars and comets by projection in two areas of 120°, four areas of 90°, or six areas of 60°. Movements of the stars with regard to the astrological houses (bound to the celestial zodiac).

The second part of the book includes all the tables connected to these different chapters.[14]

## Between Ptolemy and al-Battānī

When we examine the text of the first part of the book, we note that the name al-Battānī is not mentioned at all. The only names mentioned are those of Ptolemy, and, once, that of Theon of Alexandria.[15] He speaks of the ancients and the moderns, and he always adopts Ptolemy's position or, and it is the same thing, that of the ancients.

- He adopts Ptolemy's tropical year because, despite the different and concordant "modern" values, it is close to that of Rabbi Adda.

14 From some quotations from the book, it is clear that the two parts, the text and the table, constitute one book. We find independent tables of bar Hiyya, sometimes called *Luhot ha-Nassi* (Berlin MS 649, Bodleian MS 443 and 437, and MS Malatestiana). Apparently, from the comparison of these tables with those described in the first part of the book, we are speaking of the same thing. *Luhot ha-Nassi* does not constitute a new book. Nevertheless, Langermann (1999) already observed that the two works, the instructions of the canons (*Heshbon Mehalekhot ha-Kokhavim*) and the tables are rarely, if ever, found together in the same manuscript, a fact that indicates that they were transmitted separately. In MS Paris 1046, the title is ספר התכונה לרבי אברהם בר חייא הספרדי זצ״ל in square letters with cursive letters above ספר לוחות הנשיא. It is interesting to note that Abraham ibn Ezra was apparently the first to use this expression - לוחות הנשיא. Indeed, in MS 1046, folio 48a, we find a text from ibn Ezra referring to these tables under the name לוחות הנשיא.

15 Page 89 of the Hebrew text.

- He adopts the precession of 1° in 100 years according to the ancients (p. 101, according to the pagination of the printed Hebrew text of the first part of the book).
- He adopts an unbroken[16] direct movement of the apogee of the sun and the planets fixed to the eighth sphere. It corresponds to the precession of the equinox.
- He assumes an angle ε between the ecliptic and the equator, of 23; 51, 20° according to Ptolemy, despite the different and concordant "modern" values.
- The longitude of Jerusalem is 67.5° without any reference, and different from Ptolemy: 66° and al-Battānī: 66.5°.
- The criterion of visibility of the new moon (p. 79) is not mentioned by Ptolemy or al-Battānī.
- The necessary condition for a lunar eclipse (p. 80) is from Ptolemy.
- The necessary condition for a solar eclipse (p. 93) is from Ptolemy.
- The apogee of the sun at the epoch is roughly calculated and derived from Ptolemy, and fixed to 75.5° at the epoch (p. 66). If the apogee of the sun of Ptolemy was 65.5° in 137 CE then in 1104 CE, 967 years later, considering a movement of the fixed stars of 1° per 100 years, we should have an apogee at the epoch of 65.5 + 9.54 = 75.04°. For an unknown reason, Abraham bar Hiyya adopted 75.5°, perhaps similar to 65.5° used by Ptolemy.[17]
- The radices of the planets are given on p. 70 with explicit reference to the calculations of Ptolemy in his book.
- The position of the apogees of the planets is given according to Ptolemy on pp. 71-72.
- The maximum latitude of the planets is given according to Ptolemy on pp. 72-73.
- The mean movement in anomaly of the three superior planets is given at the bottom of p. 71.
- In ancient astronomy (Ptolemy and al-Battānī), the astronomical day is counted from noon of that day until noon of the next day. However,

16 מהלך שאין בו חילוף in contrast to the theory of trepidations championed by Thabit ibn Qura (826–901). This theory was championed by Arzachiel of Toledo (ibn al-Zarqali) (1029–87), and was still followed in the Alphonsine tables (1252).

17 In fact, in the Almagest, it seems that 65.5° is a constant value as Ptolemy found the same value as that of Hipparchus. See Pedersen (1974), p. 147 n. 10.

Abraham bar Hiyya, like al-Kwarizmi and other Arab astronomers, counts the astronomical day from noon of the preceding day until noon of today (p. 61).[18] Thus, the first astronomical day of the week, the astronomical Sunday, begins on Saturday at noon and ends on Sunday at noon.

- The equation of the days (equation of time) is explained on p. 60. The indications about the position of the minimum and maximum, and the value of the maximum, of 8 1/3°, corresponds to the indications of Ptolemy.
- The epoch of Abraham bar Hiyya is Wednesday, 29 Elul 4864 at noon or Wednesday, 21 September 1104 at noon, at the beginning of the astronomical Thursday (according to the Arabic way of counting the astronomic day, adopted by him) or the 5th day of the week.
- After all these considerations, it will be striking to note that the Hebrew text of *Sefer Heshbon Mehalekhot ha-Kokhavim* is the literal translation of passages of the *Astronomical Zij* of al-Battānī![19]

**The Astronomy of Abraham bar Hiyya Through the Astronomical Tables**

Millás Vallicrosa (1959) edited the book in Hebrew with a Spanish translation. He mentioned the titles of the different tables, but published only a small number of them, apparently those that seemed to him more original and without bearing direct similarity to Ptolemy's tables. Some of these tables were incorrectly reproduced. Furthermore, Millás Vallicrosa did not examine and comment on these tables. The understanding of these tables is far from self-evident, and the editor gave no hints as to their meaning. It would have brought some light to the subject.

**Preliminary Remarks**

In this paper we will compare Ptolemy (137 CE) to al-Battānī (880 CE). It is important to note that their astronomical models were identical. Only the parameters used to describe the model, i.e. the geometrical dimensions, the angular velocities, the inclination angle between equator and ecliptic, and the length of the periods differ. Ptolemy based himself on his own measures and on those of

18 This astronomical day fits the Arabic and Jewish civil day better, beginning at 6 p.m.

19 As the main collaborator of Plato of Tivoli, the translator of the book into Latin, he knew the book perfectly. In Nallino's new edition of al-Battānī's work, the editor refers on nearly every page of the first volume to the readings of Plato. By contrast, in the second volume – including all the tables – he does not refer to it because of the numerous misprints, but rests only on the Arabic text.

his predecessors. Al-Battānī made his own measures and adapted the model. Al-Battānī's measures had a much higher precision, but some parameters had also evolved in the meantime. Maimonides, in his *Hilkhot Kiddush ha-Hodesh*, scrupulously followed al-Battānī's model. This model is described in detail in Toomer (1984) and Pedersen (1974) for the model of Ptolemy, and for the model of al-Battānī in Nallino (1903–05), Hannover (1756) and, in a more accessible way, in Ajdler (1996) and in Ajdler (2015). The subject of the present paper is the analysis of the changing position of Abraham bar Hiyya in his astronomy and his tables.

## 1. THE EPOCH OF ABRAHAM BAR HIYYA

This moment is noon,[20] Wednesday, 29 Elul 4864 or 21 September 1104. According to bar Hiyya's assumption, this moment is the beginning of the astronomical day "Thursday, 1 Tishri 4865 or Thursday, 22 September 1104."

### 1a. The Epoch and the Jewish Calendar

**Th*e Molad* of the Year 4865**

The number of Jewish months preceding the *molad* of year 4685 is given by the fundamental formula of the Jewish calendar:[21]

$\mathbf{F_t = INT\ [(235N + 1) / 19]}$ = INT [(235 x 4864 + 1) / 19] = 60160.

The molad expressed as a part of the week is:

Mol = $[31524 + 60160 \text{ x } 765443]_{181440}$ = $[31.524 + 60160 \text{ x } 39673]_{181440}$ = 97444 hal. = 3 –18 – 244 = (4) – 18 – 244

This *molad* is thus after 3 days 18 hours and 244 *halakim*, or during the fourth day at 18h 244 *halakim*, i.e. Wednesday at 18h 244 hal or 13m 33s after noon. Tishri 1 falls on Thursday.

### 1b. The Jewish Calendar and the Julian Day

The Julian period's epoch is Monday, 1 January, −4712, at noon. At this moment, the number of elapsed days of the Julian period was 0. The Julian day no. 1 began

20 In Ajdler (2005), pp. 32-34, it was assumed that the epoch was 13m 23s or 24s before noon. However, this seems incorrect; the epoch was in fact at noon although the conjunction occurred 13m 24s before.

21 See Ajdler (2013a), pp. 7-11.

on Monday at noon and ended on Tuesday at noon. Similarly, until the 20th century, the astronomical days began at noon of the civil days of the same name.

The *molad* of *Beharad*, the beginning of the Jewish era AMI, was on Sunday, 6 October, −3760, at 23h 204 hal, Jerusalem mean time. This moment already belonged to the second Jewish day of the week, which began at 18h, hence (2) – 5 – 204. This means the second day at 5 h and 204 *halakim*. It could be written as 1 – 2 – 204, meaning 1 day 5 h and 204 hal after the beginning of the week, or 31524 hal after the beginning of the week.

Expressed in Julian days, the *molad* of *Beharad* was 347997.466203703703.

On Sunday, 6 October, −3760, at noon, 347,997 days of the JP[22] had elapsed, and on Monday, 7 October, −3760 = 1 Tishri, 1 AMI, 347,998 days of the JP had elapsed. 1 Tishri, 1 AMI began thus at 347997.25 JD, and ended at 347998.25 JD. 1 Tishri corresponded in its majority to the day 347,998 of the JP, the Julian period.[23]

### 1c. The Year 4865 and the Civil Year

Expressed in Julian days, the *molad* of 4865 is given by the following formula[24]:
Mol= 347997.466203703 + 29.530594135804 x 60160 = 2124558.00941 JD
This *molad* is thus on the civil day Wednesday, 21 September 1104, at 0h 13m 33s p.m., and Rosh ha-Shanah was on Thursday, 22 September 1104.

### 1d. The Epoch and the Arabic Calendar

From al-Battānī (1903–05), Vol. 2, p. 17, we learn that the beginning of the year Hegira 498, i.e. 1 Muharram, was on Thursday, 22 aylul "anni aerae Dhu'l qarnayn 1416," which corresponds to 22 September 1104 CE.[25]

22 Julian Period.
23 There is a second style of the Jewish calendar AMII, beginning on 1 Tishri, 2 AMI.
The *molad* of this year was *Weyad*: 6 – 14.
The first day of this year was 1 Tishri, 1 AMII = 1 Tishri, 2 AMI; it corresponds to Saturday, 27 September, −3759 or 348353 JD, beginning at 348352.25 JD and ending at 348353.25 JD.
We also note that 25 Elul, 1 AMI = Monday, 22 September, −3759 = 348348 JD.
At the time of R. Eliezer 1 Tishri, 1 AMII was still on Friday and 25 Elul, 1 AMI was still on Sunday. See Leviticus Rabbah 29,1.
24 This formula gives the same result as Shram's formula; see Ajdler (2013a), p. 57.
25 At first glance, the meaning of 1416 anni aerae Dhul qarnayn is unclear because 1416 – 311= 1105 CE! Does it correspond to the date of 1 Muharram, or is it the Roman year

### 1e. The Epoch and the Egyptian Calendar

The epoch of the Egyptian calendar is the era of Nabonassar: 26 February 747 BC at noon or 26 February – 746. This corresponds to JD 1448638. The epoch of the Egyptian years of Dhu'l qarnayn used by al-Battānī was JD 1607778. On 21 September 1104 at noon begins JD 2124558.

2124558 – 1448638 = 675920 = 1851 x 365 + 305.

The epoch of bar-Hiyya is thus the beginning (at noon) of the 306th day of the year 1852 of Nabonassar. It is also the beginning of the 306th day of the Egyptian year 1416 of the era of Dhu'l qarnayn used by al-Battānī. Indeed:

2124558 – 1607778 = 516780 = 1415 x 365 + 305.

## 2. THE MOVEMENT OF THE SUN

### 2a. The Tables of Mean Movement in Longitude of the Sun

**מהלך חמה הבינוני בשנים מחוברות ופרוטות וחדשים[26] ומהלך חמה השוה בימים, בשעות ובחלקי השעה[27]**

The table of the mean movement of the sun is organized according to days, months of 30 days, Egyptian years of 365 days, and cycles of 28 Egyptian years. We note for the longest span of time 532 Egyptian years: 57; 39, 28°.

Using the table of the sun in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 143 | 18 years | 355; 37, 25, 36° |
| Toomer, p. 143 | 1 year | 359; 45, 24, 45° |
| | 19 years | 355; 22, 50° |

corresponding to the greatest part of the Arab year? After examination of al-Battānī's conversion table, it appears that it is indeed the Roman year of 1 al-Muharram. However, in contrast to the custom adopted by al-Battānī in the entire section calculated in Roman years to consider the beginning of the Roman year on 1 March, in this table we note that the beginning of the Roman year is the preceding 1 Aylul = 1 September. Thus, for al-Battānī, we are already in the year 1416 = 1105 CE. However, according to his normal conventions to begin the civil years on 1 March, we are still in 1415 Dhu'l qarnayn = 1104 CE until the end of February. In the first part of the book *Sefer Heshbon Mehalekhot ha-Kokhavim*, chap. 8, p. 53, the text speaks of year Hegira 496 as if the epoch was the beginning of Hegira 497. Israel Eichenstein has checked MS Paris 1044, which includes the canon or the first part of the book and indeed found: Hegira 497.

26 MS Malatestiana, p. 18b and MS Paris 1046, p. 12b.

27 MS Malatestiana, p. 19a and MS Paris 1046, p. 12b.

Using the table of the sun in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 142 | 522 years | 233; 05, 22, 33° |
| Toomer, p. 143 | 10 years | 357; 34, 07, 33° |
| | 532 years | 230; 39, 30, 06° |
| | Radix[28] at the epoch | 187° |
| | Total | 57; 39, 30° versus 57; 39, 28° in the table |

of Abraham bar Hiyya. This table thus follows the tables of Ptolemy. Note that the tropical year of Ptolemy is 365d 5h 55m 12s, and that of al-Battānī is 365d 5h 46m 24s. The difference is 8m 48s. After 532 years, the difference amounts to 4681.68m = 78.03h = 3.25 days, corresponding to a difference of more than 3°!

**2b. The Equation of the Anomaly**

The anomaly of the sun is the distance of the mean sun from the apogee.[29] The equation of the anomaly is the difference between the true position and the mean position. The table of the equation of the anomaly is entitled:[30] תקון החמה במרחקה הארוך.

We find a similar table in the Almagest for an anomaly of the sun given from 3 in 3°; the maximum of the quota is 2; 23° for an anomaly of 90° until 96° and 270° until 264°. The table of bar Hiyya is given degree-by-degree. The maximum of 2; 23° is reached for 92° and 268°, and is less flat than that of Ptolemy. In any case, the curve of the quota of the anomaly can be considered as compatible with Ptolemy and in contradiction with al-Battānī, who gets a maximum of 1;59,10° for an anomaly of 92° and 268°.

## 3. THE TABLES OF MEAN MOVEMENTS OF THE MOON

**3a. Mean Motion in Longitude of the Moon**

Uniform and direct motion of the mean moon on the eccentric (or deferent), i.e.

28 This value will be discussed later.

29 In ancient astronomy; in modern astronomy we refer to the perigee.

30 MS Malatestiana, pp. 29a-30a and MS Paris 1046, folio 17b and 18a. The exact reason for the title is unclear. It probably means that the anomaly is calculated from the apogee in ancient astronomy. Note that the terminology of Bar Hiyya means: correction of the sun. Correction is the exact meaning of the term *equation*. Maimonides used the terminology: מנת המסלול, meaning the quota of the anomaly.

the motion of the center of the epicycle with respect to the center of the deferent. The table of Abraham bar Hiyya is entitled:

**מהלך לבנה השוה במרכזה[31] ובחקה[32] ובמרחקה[33] בשנים המחברות: מהלך במרכזה, מהלך החק בגלגל הקפה, המרחק מן החמה.[34] בשנים פשוטות[35] בחדשי השנה[36] בימי החדש[37] בשעות היום[38] ברגעי השעה.[39]**

We note for the longest span of time, 532 Egyptian years: 257; 0, 42°.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 182 | 522 years | 216; 06, 12, 25° |
| Toomer, p. 184 | 10 years | 213; 47, 42, 18° |
| | 532 years | 69; 53, 54, 43° |
| Radix[40] at epoch of bar Hiyya | | 187; 06, 48° |

Mean longitude of the moon: 257; 0, 43° versus 257; 0, 42° in the table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

**3b. Mean Motion of the Moon's Mean Anomaly**

Uniform and retrograde motion of the true moon on the epicycle. We note for the longest span of time, 532 Egyptian years: 29; 39, 56°.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 182 | 522 years | 231; 11, 03, 34, 43° |
| Toomer, p. 184 | 10 years | 167; 11, 14, 46, 52° |
| | 532 years | 38; 22, 18, 41, 35° |
| Radix[41]at epoch of bar Hiyya | | 351; 17, 38° |

Mean anomaly of the moon: 29; 39, 57° versus 29; 39, 56° in the table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

31 Mean motion of the moon.
32 Mean anomaly on the epicycle.
33 Mean elongation.
34 MS Malatestiana, p. 19b. In MS Paris 1046, all these tables are included in folios 13 and 14a.
35 MS Malatestiana, p. 20a.
36 MS Malatestiana, p. 20b.
37 MS Malatestiana, p. 21a.
38 MS Malatestiana, p. 21b.
39 MS Malatestiana, p. 22a.
40 This value will be discussed later.
41 This value will be discussed later.

### 3c. Mean Motion of the Moon's Elongation

Mean angular distance between sun and moon. We note for the longest span of time, 532 Egyptian years: 199; 21, 12°.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 182 | 522 years | 343; 0, 49, 51° |
| Toomer, p. 184 | 10 years | 216; 13, 34, 44° |
| | 532 years | 199; 14, 24, 35° |
| Radix[42] at epoch of bar Hiyya | | 0; 6, 48° |
| | Elongation: | 199; 21, 13° versus 199; 21, 12° in the table |

of Abraham bar Hiyya. This table follows thus the tables of Ptolemy.

### 3d. Mean Motion of the Moon in Latitude

Motion of the argument of latitude, the mean distance between the moon and the ascending node. The motion of the moon in latitude can be measured by the motion of the argument of latitude, or by the retrograde motion of the ascending node.

| | |
|---|---|
| In 1 Egyptian year: mean motion in longitude of the moon: | 129; 22, 46, 13° |
| Retrograde motion of the ascending node: | –19; 20, 1° |
| Increment of the argument of latitude: | 148; 42, 47° |

Ptolemy tabulates the increment of the argument of latitude and indicates
148; 42, 47°.

Abraham bar Hiyya, like al-Battānī, tabulates the retrograde motion of the ascending node and indicates 19; 20, 1°. With 19; 20, 1° + 129; 22, 46° =
148; 42, 47°

The tables of Abraham bar Hiyya are entitled:

מהלך ראש התלי בשנים המחברות ופשוטות והחדשים,[43] בימים ובשעות וברגע השעה.[44]

We note for the longest span of time, 532 Egyptian years 44; 59, 42°.

Using the table of the moon in the Almagest, we find for the increment of the argument of latitude:

42 This value will be discussed later.

43 MS Malatestiana, p. 22b. MS Paris 1046, pp. 14a and b.

44 MS Malatestiana, p. 23a.

| | | |
|---|---|---|
| Toomer, p. 183 | 522 years | 228; 14, 44, 50° |
| Toomer, p. 185 | 10 years | 47; 7, 52, 7° |
| Argument of latitude | 532 years | 275; 22, 36, 57° |
| Movement of moon | 532 years | 69; 53, 54, 43° |
| Retrograde motion of the ascending node | | – 205; 28, 42,14° |
| Radix[45] at the epoch of bar Hiyya | | – 199; 31, 2° |
| Ascending node | | – 44; 59, 44° versus – 44; 59, 42° in the |

table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

**3e. The Equation of the Anomaly of the Moon**

The table of Abraham bar Hiyya is entitled: תקון הלבנה, המחבר הכללי[46]. The corresponding table of the Almagest (Toomer [1984], p. 238) does not seem to be the model used by Savasorda. By contrast, his figures correspond perfectly to those of the table of al-Battānī, Vol. 2, pp. 78-83.[47] However, and this is really surprising, he did not consider the first column, Aequatio solis, as he devoted another table to the sun's equation, in which he followed Ptolemy. The difference between Ptolemy and al-Battānī concerning the equation of the solar anomaly is important but, following Ptolemy in this matter and in the choice of the value of ε, the angle of inclination of the ecliptic with regard to the equator was not a happy decision; it contradicted the direction of the history of science. In any case, the dependence of Abraham bar Hiyya, sometimes on Ptolemy, sometimes on al-Battānī, remains mysterious and unexplainable.

## 4. THE MOTION OF THE PLANETS

**4a. Saturn – שבתי: Mean Motion of Saturn**

The table of Abraham bar Hiyya is entitled: מהלך שבתי השוה בשנים מחברות

45 This value will be discussed later.

46 MS Maletestiana, pp. 30b-33a. MS Paris 1046, folios 18 and 19.

47 For a deeper understanding of the motion of the moon around the earth and the signification of the tables of al-Battānī and Abraham bar Hiyya, see Ajdler (2015) in "Luhot ha-Ibbur II," *BDD*, 30, devoted to the tables of R. Raphael Levi from Hanover in order to explain and follow Maimonides in *Hilkhot Kiddush ha-Hodesh*.

ופשוטות וחדשים,[48] בימים ובשעות וברגעי השעה[49]. We note for the longest span of time, 532 Egyptian years: 278; 56, 23°.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 427 | 522 years | 260; 34, 17, 37° |
| Toomer, p. 428 | 10 years | 122; 13, 59, 25° |
| | 532 years | 22; 48, 17, 2° |
| Radix[50] at epoch of bar Hiyya | | 256; 8, 4° |

Mean longitude of Saturn: 278; 56, 21° versus 278; 56, 21° in the table of Abraham bar Hiyya.51 This table follows thus the tables of Ptolemy.

### 4b. Jupiter – צדק: Mean Motion of Jupiter

The table of Abraham bar Hiyya is entitled: מהלך צדק השוה בשנים מחברות ופשוטות וחדשים,[52] בימים ובשעות וברגעי השעה[53]. We note for the longest span of time, 532 Egyptian years: 154; 19, 2°

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 430 | 522 years | 357; 19, 4, 4° |
| Toomer, p. 431 | 10 years | 303; 23, 48, 48° |
| | 532 years | 300; 42, 52, 52° |
| Radix[54] at epoch of bar Hiyya | | 213; 36, 12° |

Mean longitude of Jupiter: 154; 19, 5° versus 154; 19, 5° in the table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

### 4c. Mars – מאדים: Mean Motion of Mars

The table of Abraham bar Hiyya is entitled: מהלך מאדים השוה בשנים מחברות ופשוטות וחדשים,[55] בימים ובשעות וברגעי השעה[56]. We note for the longest span of time, 532 Egyptian years: 233; 50, 15°.

48 MS Malatestiana, p. 23b and MS Paris 1046, folio 14b.
49 MS Malatestiana, p. 24a and MS Paris 1046, folio 15a.
50 This value will be discussed later.
51 In MS Paris 1046 it writes incorrectly 274;56, 21°.
52 MS Malatestiana, p. 24b and MS Paris 1046, folio 15a.
53 MS Malatestiana, p. 25a and MS Paris 1046, folio 15b.
54 This value will be discussed later.
55 MS Malatestiana, p. 25b and MS Paris 1046, folio 15a.
56 MS Malatestiana, p. 26a and MS Paris 1046, folio 16a.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 433 | 522 years | 129; 5, 48, 29° |
| Toomer, p. 434 | 10 years | 112; 49, 4, 36° |
| | 532 years | 241; 54, 53, 5° |
| Radix[57] at epoch of bar Hiyya | | 351; 55, 22° |

Mean longitude of Mars: 233; 50, 15° versus 233; 50, 15° in the table of Abraham bar Hiyya. This table follows thus the tables of Ptolemy.

**4d. Venus – נוגה: Mean Motion of the Anomaly of Venus**

The table of Abraham bar Hiyya is entitled: מהלך החק לנוגה השוה בשנים מחברות ופשוטות וחדשים,[58] בימים ובשעות וברגעי השעה[59]. We note for the longest span of time, 532 Egyptian years: 10; 14, 33°.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 436 | 522 years | 103; 24, 32, 37° |
| Toomer, p. 437 | 10 years | 90; 15, 24, 46° |
| | 532 years | 193; 39, 57, 33° |
| Radix[60] at epoch of bar Hiyya | | 267; 34, 36° |

Anomaly of Venus: 101; 14, 33° versus 101; 14, 33° in the table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

**4e. Mercury – כוכב[61]: Mean Motion of the Anomaly of Mercury**

The table of Abraham bar Hiyya is entitled: מהלך החק לכותב בשנים מחברות ופשוטות וחדשים,[62] בימים ובשעות וברגעי השעה[63]. We note for the longest span of time, 532 Egyptian years: 235; 50, 15°.

57 This value will be discussed later.
58 MS Malatestiana, p. 26b and MS Paris 1046, folio 16a.
59 MS Malatestiana, p. 27a and MS Paris 1046, folio 16b.
60 This value will be discussed later.
61 Abraham bar Hiyya uses the name כותב. The signification of כותב could be “the secretary.” Indeed, the mean longitude of Mercury is equal to that of the sun. Mercury would be the secretary or the servant of the sun.
62 MS Malatestiana, p. 27b and MS Paris 1046, folio 16b.
63 MS Malatestiana, p. 28a and MS Paris 1046, folio 17a.

Using the table of the moon in the Almagest:

| | | |
|---|---|---|
| Toomer, p. 439 | 522 years | 79; 22, 7, 10, 58° |
| Toomer, p. 440 | 10 years | 179; 27, 5, 25, 29° |
| | 532 years | 258; 49, 12, 36, 27° |
| Radix[64] at epoch of bar Hiyya | | 25; 54, 9° |
| Anomaly of Mercury: | | 284; 43, 21° |

Anomaly of Mercury: 284; 43, 21° versus 284; 43, 21° in the table of Abraham bar Hiyya. This table thus follows the tables of Ptolemy.

**Conclusion**

The table of motion of the planets of Abraham bar Hiyya is deduced from the tables of Ptolemy. However, the presentation of the table is different and corresponds to that of the tables of al-Battānī. In the tables of Ptolemy, the increment of any parameter is given for a span of time and we must add to it the radix listed at the head of the table. In the tables of Abraham bar Hiyya, we have a table listing the definitive value of the parameters for years chosen from 19 to 19 after the epoch.

### 4f. Tables of Mean Motion in Longitude of the Moon, Saturn, and Mars During a Tropical Year. The Mean Motion in Anomaly of Venus and Mercury and the Mean Retrograde Motion of the Moon's Ascending Node During a Tropical Year. Difference Between the Tropical and Egyptian Years and Multiples[65]

These tables are the continuation of similar tables of motion. They allow us to check the length of Abraham bar Hiyya's tropical year. We know that the tropical year of Ptolemy is 365d 5h 55m 12s = 365.2466666d. 100 tropical years = 36500 d + 24 d + 16h = 100y +24d + 16h = 90 y + 10 y +24 d + 16h where y represents an Egyptian year of 365 days. We can then calculate with Ptolemy's tables the movements in 100 tropical years, and compare with the last entry of the tables of Abraham bar Hiyya.

**Mean Motion in Longitude of the Moon**

| | |
|---|---|
| 90 years | 124; 9, 20, 46° |
| 10 years | 213; 47, 42, 18° |

64 This value will be discussed later.

65 MS Malatestiana, pp. 61b-63a and MS Paris 1046, folio 36b.

| | |
|---|---|
| 24 days | 316; 13, 59, 25° |
| 16 hours | 8; 49, 10, 26° |
| | 303; 0, 12, 55° instead of 302; 55, 06° (Malatestiana) and 302; 58, 06° (MS Paris) |

**Mean Motion in Anomaly of the Moon**

| | |
|---|---|
| 90 years | 64; 41, 13, 2° |
| 10 years | 167; 11, 14, 47° |
| 24 days | 313; 33, 34, 31° |
| 16 hours | 8; 42, 35, 58° |
| | 194; 8, 38, 8° instead of 194; 8, 40°. |

**Double Elongation**

| | |
|---|---|
| 90 years | 146; 2, 12, 44° |
| 10 years | 216; 13, 34, 44° |
| 24 days | 292; 34, 40, 32° |
| 16 hours | 8; 7, 37, 48° |
| | (302; 58, 5, 48°) * 2 = 245; 56, 11, 36° instead of 245; 56, 12°. |

We get a very good agreement with the motions deduced from the tables of Ptolemy during 100 tropical years. We reach similar conclusions with similar calculations made for the planets. Finally, in the last table, giving the difference between the multiples of tropical years and Egyptian years, we note that the difference is a multiple of 5 h 55m 12s. Abraham bar Hiyya follows Ptolemy in these tables. As he writes clearly, in the first part of *Sefer Heshbon Mehalekhot ha-Kokhavim,*[66] he adopted the tropical year of Ptolemy because it is practically equal to the length of the year of Rabbi Adda, which underlies the Jewish calendar.[67]

## 5. THE EQUATION OF TIME

### 5a. Modern Definition

The equation of time ES = T – Tm is the difference between the true time and the mean time. It is the correction to add to (or subtract from if it is negative, thus to add algebraically to) the mean time in order to get the true time. It has thus the

66 Hebrew text, p. 46. See also *Sefer ha-Ibbur*, Book 3, chap. 1, pp. 77-78.

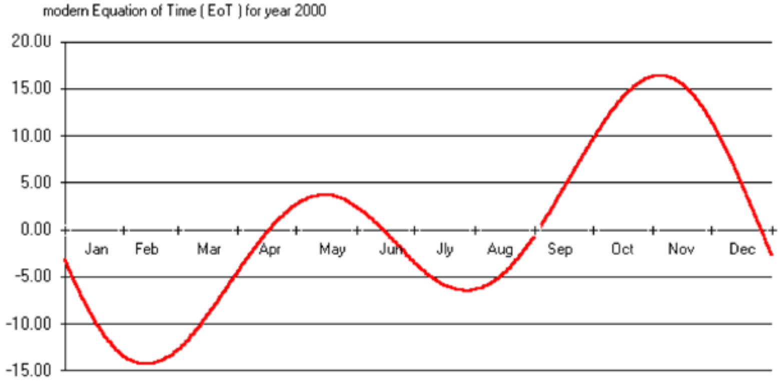
67 This can explain why he constructed his tables of the movements of the celestial bodies on the parameters of the Almagest.

same meaning as any "equation"; it is the correction to add (algebraically) to the mean value of a parameter in order to get the true value of this parameter.

We note the following specific average values of the equation of time (20th century).

On February 11, ES = – 14; 25m. At mean noon it is 11h 45m 35s true time.
On March 15, ES = 3; 47m. At mean noon it is 12h 3m 47s true time.
On July 27, ES = – 6; 20m. At mean noon it is 11h 53m 40s true time.
On November 4, ES = 16; 22m. At mean noon it is 12h 16m 22s true time.

**Figure 1: Equation of Time Es = Tm – T for Year 2000 Expressed by the Method of the Modern Astronomer**




**5b. Ancient Definition**

From 1672 onward, when Flamsteed, the astronomer royal, introduced the new equation of time, the equation of time was E = – Es = $T_m$ – T, where E is the equation of time introduced by Flamsteed, and Es is the equation of time introduced by Smart in the 20th century.

In the time of Flamsteed, the social and civil life was organized around the true time. Scholars added algebraically the equation of time to the true time for scientific reasons alone, in order to find the mean time, the time of the astronomical tables. Later, the generalization of watches and wristwatches would reverse the situation.

We note the following specific average values of the equation of time.

On February 11, E = 14; 25m. At true noon it is 12h 14m 25s mean time.

On March 15, E = – 3; 47m. At true noon it is 11h 56m 13s mean time.
On July 27, E = 6; 20m. At true noon it is 12h 6m 20s mean time.
On November 4, E = – 16; 22m. At true noon it is 11h 43m 38s mean time.

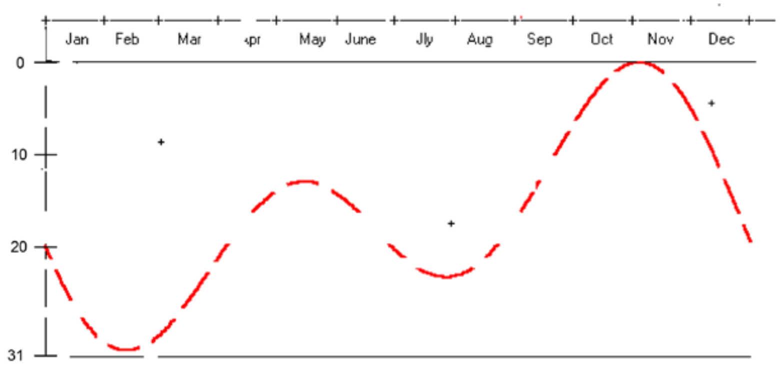
**5c. Antiquity**

Before 1672, the astronomers considered the equation of the days. Greek astronomy had already been discovered, and one of its great achievements was that the natural days or true days did not have a uniform length. They differ from an equinoctial day or mean day by a small difference dE = true day – mean day, which never exceeds 30 s in absolute value.

**Table 1: The Length of the Natural Day: True Day = 24 Mean Hours + dE**

| Date | dE in seconds | Date | dE in seconds |
|---|---|---|---|
| February 11 | 0 | July 27 | 0 |
| March 28 | –18.4 | September 17 | –21.4 |
| May 15 | 0 | November 4 | 0 |
| June 20 | 13 | December 23 | 29.9 |

**Figure 2: Equation of Time for Year 2000 Expressed by the Method of the Ancient Astronomers**




Lower x-axis = Almagest and Al-Battānī: Correction: true time to mean time = 0.
The correction from true time to mean time is subtractive from 0 to 31.2m (33.33m in Almagest).
Upper x-axis = Handy Tables. Correction: true time to mean time = 0
The correction from true time to mean time is additive from 0 to 33.33m.
The horizontal line at 1 September = modern mean time = 0.

The algebraic summation of these small differences during a certain span of time constitutes the equation of the days. It is the difference between the measure of the length of this span of time in mean days and the measure of this span of time in true days. The ancients calibrated their mean time in two ways.

1. **Almagest and al-Battānī**
   The origin of all the spans of time is chosen on about 11 February of the proleptic (fictitious) Gregorian calendar. The mean time is calibrated on the true time of this date (this date evolves slowly because the Gregorian calendar is not absolutely exact).

   - **al-Battānī**
     The correction ΔE from true time to mean time is subtractive and ranges from 0 to 31.60 minutes.[68] The mean time of al-Battānī was calibrated on the true time when L, the true longitude of the sun is 318.5°, or about 11 February (proleptic Gregorian calendar) and the maximum of the correction is reached for L = 210° (about 4 November). Thus, on about 11 February (Gregorian), the correction ΔE from true time to mean time is 0 and, on about 4 November , the correction from true time to mean time is subtractive: –31.60 m. At true noon, it is 11h 28.4m mean time. If al-Battānī had known the modern definition of the equation of time of Flamsteed, a – l, he would have found for 11 February: E = 16. 44m and for 4 November: E = –15.16m.[69] This allows us to write the important equation according to al-Battānī's model:
     **Mean Time of al-Battānī + 16.44 m = modern Mean Time.**

   - **Almagest**
     The astronomical model of Ptolemy was the same, but the parameters were different and less accurate. The correction ΔE from true time to mean time is subtractive and ranges from 0, when the true sun is in the middle of Aquarius,[70] at 315°, to 8;20° = 33.33 m, when the true

68 According to his tables: Vol. 2, p. 84: 7 ;54° = 7.90° =31.60m = 31m 36s. But, according to the main text, Vol. 1 p. 49: 7 ;48° = 7.80° = 31.20m 31m 12s.

69 Note that the curve of the equation of time evolves in function of the time; see Meeus (1991), chap. 27. It depends also on the precision of astronomical parameters (obliquity of the ecliptic, eccentricity of the earth's orbit) used by Ptolemy and al-Battānī. This explains the apparent contradiction between the extrema of Ptolemy, al-Battānī, and the moderns.

70 At this sun's longitude, a - l the equation of time of Flamsteed is 4;48° = 19.20m.

longitude of the sun L = 210° (the beginning of Scorpio).[71] Practically, the mean time of the Almagest was calibrated on the true time at the epoch, on Nabonassar 1, Toth 1 (26 February – 746), when l, the sun's mean longitude was 330;45°, L was 333;08°(near to 315°), and Alfa was 335;08°.[72] The equation of time of Flamsteed, a – l was 4;23° = 17.53m. Thus, at the epoch, the correction from true time to mean time is 0 and at the beginning of Scorpio the correction from true time to mean time is subtractive and is about – 7;55° or –31.67 m.[73] If Ptolemy had known the modern definition of Flamsteed's equation of time he would have found at the epoch: E = 17.53m and at the beginning of Scorpio: E = –14.14m. This allows us to write, according to Ptolemy's model, the important equation:

**Mean Time of Almagest + 17.53 m = modern Mean Time.**

2. **Handy Tables**

In another set of tables, Ptolemy adopted another epoch on Philip 1, Toth 1 (– 323, 12 November). At this epoch, the mean longitude of the sun l was 227°;40 or 17°40' in Scorpio, close to its beginning. This was similar to the system adopted in the "Connoissance des Temps" for the equation of the clocks, where the origin of the equation of time was fixed on about 4 November, when the sun is at the beginning of Scorpio. In the Handy Tables, Flamsteed's equation of time for the Era of Philip with respect to the era of Nabonassar is 7;38° = 30.53m.[74] Thus a - l, Flamsteed's equation of time at the epoch of Philip is 4;23° – 7;38° = – 3;15° or –13m. This allows us to write, according to Ptolemy's model, the important equations:

**Mean Time of Handy tables – 13m = modern Mean Time.**

**Mean Time of Almagest + 30.53m = Mean Time of Handy tables.**[75]

For a complete theoretical theory and additional details see Delambre (1817),

71 At this sun's longitude, a - l the equation of time of Flamsteed is 4;48° – 8;20° = – 3; 32° = – 14.13m.

72 See Neugebauer (1975), p. 67.

73 Exactly 4;23° + 3;32° = 7 ;55° = 31.67m.

74 See Neugebauer (1975), Vol. 2, p. 984.

75 At the epoch of the Almagest L=330°;45 near to 315°. At the epoch of the Handy tables L= 227° ;40 near to 210°. Therefore, the maximum difference of time of 33.33m is reduced to about 30.53m between both Eras. Note that this equation is often written: Mean Time Almagest + ~ 32m = Mean Time Handy tables (Neugebauer, p. 985).

Lalande (1792), Smart (1977), Neugebauer (1975), Danjon (1980) and, in a simplified way, Ajdler (2005).

Ptolemy studied the equation of the days in Book III, Chapter 9. He did not tabulate it, but indicated that it is zero in the middle of Aquarius, 315°, and is maximum at the end of Libra and beginning of Scorpius, 210°, and its maximum value is 8; 20° = 33.33m.[76] This value is connected to the maximum value of the equation of the anomaly of the sun of 2; 23°. Abraham bar Hiyya adopted the same equation of the sun, in contrast to the more exact value of 1; 59° adopted by al-Battānī. It is thus normal that Abraham bar Hiyya also followed Ptolemy here in the treatment of the equation of time.

However, we will see later that there is a doubt whether Abraham Bar Hiyya calibrated his mean time according to the Almagest (subtractive correction from true time to mean time) or according to the Handy Tables (additive correction from true time to mean time).

Abraham by Hiyya gives a detailed table of the equation of the days, in function of the true longitude of the sun degree-by-degree, compatible with the principles developed by Ptolemy. The equation is 0 for 315 – 317° and it reaches its maximum for a longitude of 210 – 218°; its maximum is 8; 20° corresponding to 33m 20s. He follows Ptolemy's model. The title of the table is ערך החלוף שבין הימים ולילותיהן ובין הבאות אחריהן.[77] This title is misleading and provides no clear explanation.

## 6. THE DECLINATION OF THE POINTS OF THE ECLIPTIC

This table is entitled: קשת הנמיכות בין אופן המזלות ואופן המישור[78]. We note that the maximum declination is 23; 51, 20°, and it corresponds to the value of Ptolemy.

## 7. CHECKING THE RADICES OF ABRAHAM BAR HIYYA

### 7a. Radices at the Epoch of Abraham bar Hiyya according to Ptolemy

We have seen that the epoch of Abraham bar Hiyya is the epoch of Nabonassar + 1851 years of 365 days + 305 days. Using the tables of Ptolemy, we can refer

76 This is the value given by Ptolemy. It differs slightly from the modern calculations mentioned above. See former note.

77 MS Malatestiana, pp. 12a and 12b, and MS Paris 1046, folio 9b.

78 MS Malatestiana. pp. 8b-9b, and MS Paris 1046, folio 7b.

to the entries 810 + 810 + 216 + 15 + 10 months + 5 days. Normally, we would finally subtract 22m because, according to the values adopted by Ptolemy, when it is noon in Jerusalem it is only 11h 38m in Alexandria.[79] We refer to the tables of Ptolemy in the Almagest: Toomer (1984), pp. 142-43 and pp. 182-87.

## Mean Longitude of the Sun and Moon According to the Almagest

**Table 2: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Sun and Moon According to Ptolemy**

| | Sun's mean longitude | | | Moon's mean longitude | | | Moon's anomaly | | | Argument of latitude | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | deg. | min | sec | deg | min | sec | deg | min | sec | deg | min | sec |
| radix | 330 | 45 | | 41 | 22 | | 268 | 49 | | 354 | 15 | |
| 810y | 163 | 4 | 12 | 37 | 24 | 7 | 222 | 10 | 57 | 217 | 37 | 22 |
| 810y | 163 | 4 | 12 | 37 | 24 | 7 | 222 | 10 | 57 | 217 | 37 | 22 |
| 216y | 307 | 29 | 7 | 225 | 58 | 26 | 83 | 14 | 55 | 82 | 1 | 58 |
| 15y | 356 | 21 | 11 | 140 | 41 | 33 | 250 | 46 | 52 | 70 | 41 | 48 |
| 10m | 295 | 41 | 26 | 352 | 54 | 53 | 319 | 29 | 41 | 8 | 48 | 19 |
| 5d | 4 | 55 | 41 | 65 | 52 | 55 | 65 | 19 | 30 | 66 | 8 | 48 |
| **Alexandria** | **181** | **20** | **49** | **181** | **38** | **1** | **352** | **1** | **52** | **297** | **10** | **37** |
| –22mn | | | 54 | | 12 | 5 | | 11 | 58 | | 11 | 10 |
| **Jerusalem** | **181** | **19** | **55** | **181** | **25** | **56** | **351** | **49** | **54** | **296** | **59** | **27** |

## The Superior Planets: Saturn, Jupiter, and Mars

We refer to the tables of Ptolemy, Toomer (1984), pp. 427-41.

## Mean Longitude of the Superior Planets According to the Almagest

**Table 3: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Mean Position in Longitude of the Superior Planets According to Ptolemy**

| | Saturn | | | Jupiter | | | Mars | | |
|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec | deg | min | sec |
| radix | 296 | 43 | | 184 | 41 | | 3 | 32 | |
| 810y | 180 | 53 | 13 | 95 | 8 | 54 | 138 | 15 | 13 |
| 810y | 180 | 53 | 13 | 95 | 8 | 54 | 138 | 15 | 13 |
| 216y | 120 | 14 | 11 | 73 | 22 | 22 | 276 | 52 | 3 |

79 There is a difference of 396 hal = 22m between Alexandria and Jerusalem.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 15y | 183 | 20 | 59 | 95 | 5 | 43 | 349 | 13 | 37 |
| 10m | 2 | 47 | 37 | 24 | 56 | 12 | 157 | 13 | 4 |
| 5d | 0 | 10 | 3 | 0 | 24 | 56 | 2 | 37 | 13 |
| **Alexandria** | **245** | **2** | **16** | **208** | **48** | **1** | **345** | **58** | **19** |
| –22min | | | 2 | | | 4 | | | 27 |
| **Jerusalem** | **245** | **2** | **14** | **208** | **47** | **57** | **345** | **57** | **52** |

## Mean Anomaly of the Inferior Planets According to the Almagest

**Table 4: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Mean Anomaly of the Inferior Planets According to Ptolemy**

| | Venus | | | Mercury | | |
|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec |
| Radix | 71 | 7 | | 21 | 55 | |
| 810y | 110 | 48 | 26 | 135 | 34 | 19 |
| 810y | 110 | 48 | 26 | 135 | 34 | 19 |
| 216y | 5 | 32 | 55 | 132 | 9 | 9 |
| 15y | 135 | 23 | 7 | 89 | 10 | 38 |
| 10m | 184 | 57 | 9 | 212 | 0 | 35 |
| 5d | 3 | 4 | 57 | 15 | 32 | 1 |
| **Alexandria** | **261** | **42** | **0** | **21** | **56** | **1** |
| –22min | | | 34 | | 2 | 51 |
| **Jerusalem** | **261** | **41** | **26** | **21** | **53** | **10** |

Obviously, Abraham bar Hiyya did not establish his radices on the basis of Ptolemy’s tables. This is surprising as the other elements of the tables, the motions of the sun, moon, and planets, even motions during long spans of time as long as 532 Egyptian years, are based rigorously on Ptolemy’s tables.

### 7b. Radices at the Epoch of Abraham bar Hiyya According to al-Battānī

Calculation according to al-Battānī’s tables corresponding to the Roman calendar (Julian calendar). The epoch is preceded by 1415 Egyptian years from Dhu’l qarnayn + 305 days. We refer to the tables of al-Battānī, Nallino (1903–05), Vol. 2, pp. 72-77 and pp. 102-105.

### Mean Longitude of the Sun and Moon According to al-Battānī (Roman Calendar)

**Table 5: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Sun and Moon According to al-Battānī; see above the Epoch and the Egyptian Calendar**

| | Sun's mean longitude | | | Moon's mean longitude | | | Moon's anomaly | | | Ascending node | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec | deg | min | sec | deg | min | sec |
| 1411 | 344 | 51 | 42 | 195 | 11 | 22 | 185 | 1 | 51 | 111 | 15 | 17 |
| 4 | 0 | 2 | 14 | 170 | 43 | 7 | 7 | 56 | 23 | 77 | 21 | 41 |
| August | 181 | 21 | 36 | 264 | 27 | 27 | 243 | 57 | 25 | 9 | 44 | 34 |
| 21d | 20 | 41 | 55 | 276 | 42 | 16 | 274 | 21 | 53 | 1 | 6 | 44 |
| **Ar-Raqqah** | **186** | **57** | **27** | **187** | **4** | **12** | **351** | **17** | **32** | **199** | **28** | **16** |
| +27m | | 1 | 7 | | 14 | 49 | | 14 | 42 | | | 4 |
| **Jerusalem** | **186** | **58** | **34** | **187** | **19** | **1** | **351** | **32** | **14** | **199** | **28** | **20** |

### Mean Longitude of the Superior Planets and Mean Anomaly of the Inferior Planets According to al-Battānī (Roman Calendar)

**Table 6: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Longitude of the Three Superior Planets and of the Mean Anomaly of the Two Inferior Planets According to al-Battānī**

| | Saturn | | Jupiter | | Mars | | Venus | | Mercury | |
|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | deg | min | deg | min | deg | min | deg | min |
| 1411 | 200 | 20 | 75 | 5 | 198 | 50 | 320 | 23 | 250 | 5 |
| 4 | 48 | 56 | 121 | 28 | 45 | 40 | 180 | 45 | 218 | 52 |
| August | 6 | 9 | 15 | 17 | 96 | 26 | 113 | 26 | 211 | 28 |
| 21d | 0 | 42 | 1 | 45 | 11 | 0 | 12 | 57 | 65 | 14 |
| **Ar-Raqqah** | **256** | **7** | **213** | **35** | **351** | **56** | **267** | **31** | **25** | **39** |
| +27m | | 0 | | 0 | | 0 | | 1 | | 4 |
| **Jerusalem** | **256** | **7** | **213** | **35** | **351** | **56** | **267** | **32** | **25** | **43** |

### Calculation According to al-Battānī's Tables Corresponding to the Arabic Calendar

We refer to the tables of al-Battānī, Vol. 2, pp. 18-23 and pp. 24-28.

## Mean Longitude of the Sun and Moon According to al-Battānī (Arabic Calendar)

**Table 7: Calculation of the Radices at the Epoch of Abraham bar Hiyya of the Sun and Moon According to al-Battānī**

| | Sun's mean longitude | | | Moon's mean longitude | | | Moon's anomaly | | | Ascending node | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec | deg | min | sec | deg | min | sec |
| 481 | 9 | 23 | 26 | 12 | 26 | 39 | 127 | 50 | 13 | 240 | 29 | 49 |
| 17 | 177 | 33 | 58 | 174 | 37 | 33 | 223 | 27 | 24 | 318 | 58 | 34 |
| **Ar-Raqqah** | **186** | **57** | **24** | **187** | **4** | **12** | **351** | **17** | **37** | **199** | **28** | **23** |
| +27m | | 1 | 7 | | 14 | 49 | | 14 | 42 | | | 4 |
| **Jerusalem** | **186** | **58** | **31** | **187** | **19** | **1** | **351** | **32** | **19** | **199** | **28** | **27** |

See above the Epoch and the Arabic Calendar.

## Mean Longitude of the Superior Planets and Mean Anomaly of the Inferior Planets According to al-Battānī (Arabic Calendar)

**Table 8: Calculation of the Radices at the Epoch of Abraham bar Hiyya, of the Longitude of the Three Superior Planets and of the Mean Anomaly of the Two Inferior Planets According to al-Battānī**

| | Saturn | | Jupiter | | Mars | | Venus | | Mercury | |
|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | deg | min | deg | min | deg | min | deg | min |
| 481 | 54 | 20 | 72 | 48 | 74 | 52 | 153 | 36 | 30 | 59 |
| 17 | 201 | 48 | 140 | 48 | 277 | 1 | 113 | 57 | 354 | 47 |
| **Ar-Raqqah** | **256** | **8** | **213** | **36** | **351** | **53** | **267** | **33** | **25** | **46** |
| +27m | | 0 | | 0 | | 0 | | 1 | | 4 |
| **Jerusalem** | **256** | **8** | **213** | **36** | **351** | **53** | **267** | **34** | **25** | **50** |

## Radices Adopted by Abraham bar Hiyya

**Table 9: Comparison of the Radices at the Epoch of Abraham bar Hiyya According to al-Battānī with the Radices Adopted by Abraham bar Hiyya in His Tables**

| Radices at noon at the epoch on 21-9-1104 | Ptolemy in Alexandria | al-Battānī in ar-Raqqah | Tables of Abraham bar Hiyya | Difference Abr. bar Hiyya – al-Battānī |
|---|---|---|---|---|
| Long. sun | 181; 20, 49° | 186; 57, 24° | 187° | 0; 2, 36° |
| Long. moon | 181; 38, 1° | 187; 04, 12° | 187; 6, 48° | 0; 2, 36° |
| Elongation | 0; 17, 12° | 0; 6, 48° | 0; 6, 48° | 0 |
| Anom. moon | 352; 1, 52° | 351; 17, 37° | 351; 17, 38° | 0; 0, 1° |
| Ascen. node | 244; 27, 24° | 199; 28, 23° | 199; 31, 2° | 0; 2, 39° |
| Long. Saturn | 245; 2, 16° | 256; 8° | 256; 8, 4° | 0; 0, 4° |
| Long. Jupiter | 208; 48, 1° | 213; 36° | 213; 36, 12° | 0; 0, 12° |
| Long. Mars | 345; 58, 19° | 351; 53° | 351; 55, 22° | 0; 2, 22° |
| Anom. Venus | 261; 42, 0° | 267; 33° | 267; 34, 36° | 0; 1, 36° |
| Anom. Mercury | 12; 56, 1° | 25; 46° | 25; 54, 9° | 0; 8, 9° |

It is clear that Abraham bar Hiyya did not use the radices calculated by the tables of Ptolemy. Indeed, the difference for the longitude of the sun reaches 5.5° and would be inacceptable. He clearly used the values calculated with al-Battānī's tables. In the case of the planets too, he used al-Battānī's values. Of course, we must take into consideration the precision of the calculations. In the case of the sun and moon, he obviously rounded off the mean longitude of the sun, but adapted the mean longitude of the moon in order to maintain the exact value of the elongation.[80] The examination of the longitude of the moon at noon in ar-Raqqah with regard to the longitude of the moon at noon in Jerusalem allows us to conclude that, despite his statement, he used the data given by al-Battānī for ar-Raqqah,[81] and did not take the difference of 27m[82] between Jerusalem and ar-Raqqah into account. Henceforth, we will assume that Abraham bar Hiyya

80 This cannot explain the rounding off of the radices according to Ptolemy as we observe the precision of the radices according to al-Battānī.

81 See below in chap. 9: Radices according to al-Battānī, the proof that bar Hiyya considered the radices of al-Battānī in ar-Raqqah. For the sun: 187°- 0; 2, 36° = 186; 57, 24°. Ascending node: 199; 31, 2° - 0; 2.39° = 199; 28, 23°.

82 According to al-Battānī: ar-Raqqah: 73°; 25' and Jerusalem 66°; 30', hence a difference of 6°; 45' corresponding to 27m.

neglected the difference of longitude between ar-Raqqah and Jerusalem, and privileged the tables of al-Battānī in the Arabic calendar.[83]

The question that arises then is why he used Ptolemy's tables instead of those of al-Battānī in his tables for the movement of the sun, moon, and planets?

Apparently, he considered that Ptolemy's tables lose their precision in the long run, but, in the short run, it seems he had more confidence in the tables of Ptolemy. Therefore, he used the radices deduced from al-Battānī, about 220 years after their redaction, instead of the tables of Ptolemy established about 1000 years earlier but preferred, wrongly, in the short run,[84] the increment of the parameters given by Ptolemy.

## 8. TABLES OF MEAN CONJUNCTION AND OPPOSITION AT THE END OF THE CYCLES AFTER THE EPOCH

### 8a. Tables of the Mean Conjunction at the End of the Jewish Cycles Following the Epoch

This table is entitled: לוח חבורי המאורות בחדש תשרי בראש כל מחזור ומחזור מראש מחזור רנ״ז ולהלן[85]

83 The differences between the tables calculated in the Arabic and Roman calendars are small but not negligible, and we can ascertain that Abraham bar Hiyya worked with the tables in the Arabic calendar. We will see later that the time of the mean conjunction, slightly preceding the epoch, also proves, without any doubt, that Abraham bar Hiyya worked with the results of al-Battānī's tables without taking into account the difference of longitude Jerusalem–ar-Raqqah.

84 Things are relative: his tables extend to a 532-year time span, more than half the time span of 950 years since the redaction of Ptolemy's tables. If the tables of Ptolemy lead to an error of more than 5.5° on the position of the sun and the moon, then after 532 years we certainly have an error of more than 3.08° on the position of the sun and the moon.

85 MS Malatestiana, p. 75b, and MS Paris 1046, folio 44b, at the end of the manuscript. The whole table is concentrated on one sheet; this creates some confusion in the reading. In MS Berlin, the table is distributed over two sheets. We adopted this configuration.

**Table 10: First Part of the Table: לוח חבורי המאורות**

| Day | Complete cycles | A.M | Egyptian years | Months | Days | Hours | min | sec |
|---|---|---|---|---|---|---|---|---|
| Wednesday | *256* | *4864* | *0* | 13m 24s[86] before the epoch which was at noon *of Wednesday* | | | | |
| | שורש | | | | | | | |
| Friday | 257 | 4883 | 19 | 0 | 4 | 16 | 19 | 40 |
| Monday | 258 | 4902 | 38 | 0 | 9 | 8 | 52 | 43 |
| Thursday | 259 | 4921 | 57 | 0 | 14 | 1 | 28 | 47 |
| Saturday | 260 | 4940 | 76 | 0 | 18 | 17 | 58 | 50 |
| Tuesday | 261 | 4959 | 95 | 0 | 23 | 10 | 31 | 53 |
| Friday | 262 | 4978 | 114 | 0 | 28 | 3 | 4 | 57 |
| Sunday | 263 | 4997 | 133 | 1 | 2 | 19 | 38 | 0 |
| Wednesday | 264 | 5016 | 152 | 1 | 7 | 12 | 11 | 3 |
| Saturday | 265 | 5035 | 171 | 1 | 12 | 4 | 44 | 7 |
| Monday | 266 | 5054 | 190 | 1 | 16 | 21 | 17 | 10 |
| Thursday | 267 | 5073 | 209 | 1 | 21 | 13 | 50 | 13 |
| Sunday | 268 | 5092 | 228 | 1 | 26 | 6 | 23 | 17 |
| Tuesday | 269 | 5111 | 247 | 2 | 0 | 22 | 56 | 20 |
| Friday | 270 | 5130 | 266 | 2 | 5 | 15 | 29 | 23 |
| Monday | 271 | 5149 | 285 | 2 | 10 | 8 | 2 | 27 |
| Thursday | 272 | 5168 | 304 | 2 | 15 | 0 | 35 | 30 |
| Saturday | 273 | 5187 | 323 | 2 | 19 | 17 | 8 | 33 |
| Tuesday | 274 | 5206 | 342 | 2 | 24 | 9 | 41 | 37 |
| Friday | 275 | 5225 | 361 | 2 | 29 | 2 | 14 | 40 |

In the present table, the text in italics was added in order to improve the understanding. The transcription by Millás-Vallicrosa, p. 126, was mistaken. In contrast with the title, we find on each row the situation at the end of that cycle or at the beginning of next cycle, i.e. the beginning of the cycle following the end of the indicated cycle and the indicated year.

The left-hand column gives the Jewish[87] day when the mean conjunction occurs.

86 And not 13m 200hal as is erroneously written in Millás Vallicrosa (1959), p. 126. In MS Paris 1046, the reading is 33s, which is certainly incorrect.

87 The *molad* of Tishri 4922 was 5 – 19 – 949. The mean conjunction was thus on Thursday, slightly after noon. This was thus on the Jewish Thursday, but it was already the "astronomic Friday" Arabic style adopted by Abraham bar Hiyya. The column gives thus a day later, the astronomical day of the Roman style.

There is a problem on the fourth row. The *molad* of Tishri 4941 is 1 – 12 – 464 and the mean conjunction is slightly before 6 a.m. on Sunday morning. Obviously, he is considering here astronomic days of the Roman style beginning at noon of this day. The next column gives the last year of the cycle in Anno Mundi of *Beharad*, at the end of which we consider the conjunction. The next columns give the number of Egyptian years, months, days, minutes, and seconds elapsed since the epoch until the considered mean conjunction. The length of a cycle is 235 * (29 – 12 – 793) = 6939d 16h 595 hal = 6939d 16h 33m 3.33 sec = 6935d + 4d + 16h +33m +3.33s = 19 Egyptian years + 4d + 16h +33m +3.33s. Practically, 3 * 3.33s = 3s + 3s +4s.

As the conjunction of Tishri 4884 was 6935d + 4d + 16h +19m + 40s, days after the epoch, followed by spans ending with 3s and 4s, the first conjunction of Tishri 4865 must be: 13m 23s before the epoch. However, two manuscripts write 13m 24s while MS Paris gives 33s.

**Table 11: Second Part of the Table: לוח חבורי המאורות**

| Completed cycles | Common position of sun and moon | | | Moon’s anomaly | | | Longitude of moon’s ascending node | | |
|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec | deg | min | sec |
| *256* | 186 | 59 | 27 | 351 | 10 | 19 | 199 | 31 | 0 |
| **שורש** | | | | | | | | | |
| 257 | 186 | 59 | 37 | 298 | 5 | 52 | 207 | 6 | 14 |
| 258 | 186 | 59 | 48 | 245 | 1 | 25 | 214 | 41 | 27 |
| 259 | 186 | 59 | 58 | 191 | 56 | 58 | 222 | 16 | 40 |
| 260 | 187 | 0 | 9 | 138 | 52 | 32 | 229 | 51 | 53 |
| 261 | 187 | 0 | 19 | 84 | 48 | 5 | 237 | 27 | 6 |
| 262 | 187 | 0 | 30 | 32 | 43 | 38 | 245 | 2 | 19 |
| 263 | 187 | 0 | 40 | 339 | 39 | 12 | 252 | 37 | 33 |
| 264 | 187 | 0 | 51 | 286 | 38 | 45 | 260 | 12 | 44 |
| 265 | 187 | 1 | 1 | 233 | 30 | 18 | 267 | 47 | 57 |
| 266 | 187 | 1 | 12 | 180 | 25 | 51 | 275 | 23 | 10 |
| 267 | 187 | 1 | 22 | 127 | 21 | 25 | 282 | 58 | 52 |
| 268 | 187 | 1 | 33 | 74[88] | 16 | 58 | 290 | 33 | 35 |
| 269 | 187 | 1 | 43 | 21 | 12 | 31 | 298 | 8 | 48 |

88 And not 84, as wrongly adopted by Millás Vallicrosa.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 270 | 187 | 1 | 54 | 328 | 8 | 4 | 305 | 44 | 1 |
| 271 | 187 | 2 | 4 | 275 | 3 | 35 | 313 | 19 | 14 |
| 272 | 187 | 2 | 15 | 221 | 59 | 11 | 320 | 54 | 26 |
| 273 | 187 | 2 | 25 | 168 | 54 | 44 | 328 | 29 | 39 |
| 274 | 187 | 2 | 35 | 115[89] | 50 | 18 | 336 | 4 | 52 |
| 275 | 187 | 2 | 46 | 62 | 45 | 51 | 343 | 40 | 5 |

In the present table the text in italics was added in order to improve the understanding. The transcription by Millás Vallicrosa, p. 127, was seriously mistaken, not to say faked. In the transcription of this table, which he clearly did not understand correctly, the eight last rows are correct. Then, facing cycle 267, a line of the table was skipped and we find, facing 267, the data of cycle 266 and the situation repeats itself until the beginning of the table. The data of the first row, which concern the conjunction at the end of the year 4864 or the beginning of 4865, are thus facing the cycle 257. The table was completely cooked! In contrast with the title, we find the situation at the end of that cycle or at the beginning of next cycle on each row.

We note that that common longitude of sun and moon increases by about 10.49” after a cycle of 19 Jewish years. Indeed, let us consider Ptolemy’s figures. The year of Ptolemy is 365d 5h 55m 12s = 365.2466666d.

The angular velocity of the sun is 360°/365.24666666 = 0.985635278444°/d and the length of a cycle is 6939.689621913579 d. By multiplication, we find the angular motion of the sun during 19 Jewish years: 6840.002912° = 19 * 360°+ 0.002912°= 19 * 360 + 10.49”.

The existence of this remainder results from the fact that the year of Ptolemy of 365d 5h 55m and 12s is slightly shorter than the average Jewish year of 365d 5h 55m 25.4386s. The difference is 13.4386s and, after 19 years, it amounts to 4m 15.3334s = 4.2555m. If we multiply the angular velocity of the sun by the length of 19 years of Ptolemy, 6939d 16h 28m 48s, we find exactly 6840°, without any remainder. Similarly, 0.985635278444°/d * 4.2555m / (60*24) = 0.00291° = 10.49”. These 10.49” represent the movement of the sun during these 4.26m between the end of the 19 years of Ptolemy and the end of the cycle of 19 Jewish years.

The same calculation with the data of al-Battānī gives:
Year of al-Battānī: 365d 5h 46m 24s = 365.240555555°/d.
The angular velocity is 360°/365.240555555 = 0.98565176984 °/d.

89 As in MS Berlin, MS Paris, and Millás Vallicrosa, and not 105 as in MS Malatestiana, p. 75b.

The angular movement of the sun during 19 Jewish years is then: 6840.1173°. The increment of the common longitude is then 0.1173° = 0; 7, 2.5°.

Therefore, the radices are certainly derived from al-Battānī's tables but the calculation of the tables, like the present tables 11 and 12, is performed with the data and the tables of Ptolemy.

**Table 12. Calculation According to the Tables of Ptolemy of the Movement of the Four Parameters of Sun and Moon During a Period of 19 Jewish Years Equal to 235 Lunations or 6939 Days 16 Hours and 595 *Halakim***

| | Sun's mean longitude | | | | Moon's mean longitude | | | | Moon's anomaly | | | | Argument of latitude | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | III | deg | min | sec | III | deg | min | sec | III | deg | min | sec | III |
| 18y | 355 | 37 | 25 | 36 | 168 | 49 | 52 | 10 | 156 | 56 | 14 | 36 | 156 | 50 | 9 | 49 |
| 1y | 359 | 45 | 24 | 45 | 129 | 22 | 46 | 14 | 88 | 43 | 7 | 29 | 148 | 42 | 47 | 13 |
| 4d | 3 | 56 | 33 | 9 | 52 | 42 | 19 | 54 | 52 | 15 | 35 | 45 | 52 | 55 | 2 | 39 |
| 16h | 0 | 39 | 25 | 31 | 8 | 47 | 3 | 19 | 8 | 42 | 35 | 58 | 8 | 49 | 10 | 27 |
| 595 | 0 | 1 | 21 | 27 | 0 | 18 | 8 | 53 | 0 | 18 | 0 | 0 | 0 | 18 | 13 | 2 |
| total | 0 | 0 | 10 | 28 | 0 | 0 | 10 | 30 | 306 | 55 | 33 | 48 | 7 | 35 | 23 | 10 |

We see thus that at the end of a cycle of 19 Jewish years the common mean longitude of sun and moon increases by about 10.5'', the moon's anomaly increases by 306; 55, 33, 48°, and the argument of latitude increases by 7; 35, 23°. The longitude of the ascending node diminishes by – 7; 35, 13°. We note the exceptional precision of Ptolemy's table. The conjunction occurs at the end of the 19 Jewish years and the common longitude is indeed 0; 0, 10, 30°.

**Table 13: The Mean Conjunction After 19 Egyptian Years Occurs 4d 16h 595 hal or 4d and 41' 23'' After the End of the 6935 Days of the 19 Egyptian Years**

| 19 Egyptian Years | Sun and Moon's mean longitude | Moon's anomaly | Ascending node |
|---|---|---|---|
| Ptolemy | 0; 0, 10, 30° | 306; 55, 34° | 7; 35, 23° |
| al-Battānī | 0; 0, 7, 2° | 306; 55, 33° | 7; 34, 48° |

Because of a slight difference between Ptolemy and al-Battānī, in the angular velocity of the sun and moon and ascending node, their evolution is slightly divergent. Abraham bar Hiyya clearly adopted the values of Ptolemy. See Ptolemy: Toomer (1984), p. 280, and al-Battānī (1903–05), p. 32 and p. 86.

This can also be demonstrated, together with a justification of the variation of the common position of sun and moon, the evolution of the moon's anomaly, and the ascending node, during a cycle of 19 Jewish years or 235 mean lunations, by using Ptolemy's tables (Toomer [1984], pp. 142-43 and 182-87). This enables us to construct Table 12 according to Ptolemy's table of mean conjunctions (Toomer [1984], p. 280) and it confirms these results.

After 19 Egyptian years or 6935 days, the mean conjunction occurs 4d and 41' 23''of a day later. Indeed, 16h 595 hal = 0; 41, 23° of a day. The increase of longitude of sun and moon and the increase of the sun's anomaly is 0; 0, 10°, the increase in the moon's anomaly is 306; 55, 34°, and the increase in the argument of latitude is 7; 35, 23°. It corresponds to an increase of the absolute value of the longitude of the ascending node of 7; 35, 23°. In fact, – 7; 35, 23°, because this movement is retrograde. It appears clearly that this table was constructed according to the tables of Ptolemy, and we got the variation of the three parameters, sun, and moon's mean longitude, moon's anomaly and the mean longitude of the ascending node after 19 Jewish years, enabling checking the different figures of the table.

Indeed, the preceding numbers represent the difference between the successive rows of the table.

**Calculation of the Mean Conjunction Preceding the Epoch**

From Table 5 (Roman calendar) we find at the epoch: l = 186; 57, 27° and l' = 187; 4, 12°. Elongation: 0; 6, 45° = 405''. In one hour the variation of the elongation sun–moon is 1976 – 148 = 1826 ''/h. The mean conjunction was 405 / 1826 = 0.221796h = 13m 18s. From Table 7 (Arabic calendar) we find at the epoch: l = 186; 57, 24° and l' = 187; 4, 12°. Elongation: 0°; 6, 48° = 408''. The mean conjunction was thus 408 / 1826 = 0.223439h = 13m 24s before the epoch, which was at noon.

The hourly movement of the sun is 2' 28'' = 148''/h and that of the moon is 32' 56'' = 1976''/h. During the span of time of 13m 24s the movement of the sun is 148 *(13.40 / 60) = 33'' and the movement of the moon 1976 * (13.40 / 60) = 441.3'' = 0; 7, 21°.

The common position of sun and moon at the moment of the conjunction preceding the epoch by 13m 24s, is 186°; 59', 27''. 13m 24s later, at the moment of the epoch, the mean longitude of the sun is 186; 59, 27° + 0; 0, 33° = 187°, and that of the moon is 186; 59, 27°+ 0; 7, 21° = 187; 6, 48°. The table is thus fully coherent.

On the third row of Table 10, we find the span of time between the epoch and the first conjunction of Tishri 4884. It is 19 Egyptian years + 4d 16h 595 hal – 13m

24s = 19years + 4d 16h 33.06m – 13.40m = 19y + 4d 16h 19m 39.6s. Then, at the end of each cycle, we must add an additional 4d 16h 33m 3.33s.

**Conclusion**

- Abraham bar Hiyya used the Arabic tables of al-Battānī when he calculated his radices.
- We have the justification of the origin of the span of time of 13m 24s before the epoch for the mean conjunction.
- We check that the longitude mentioned on the first row of the table, at the moment of the first mean conjunction, is coherent with the radices adopted by Savasorda at noon.
- Apart from the radices, which were calculated with al-Battānī's tables, all the other tables were calculated with Ptolemy's data and tables.

### 8b. Tables of the Mean Opposition After the End of the Jewish Cycles Following the Epoch

This table is entitled: לוח ניגודי המאורות במחצית תשרי שבראש כל מחזור ממחצית חדש תשרי מראש מחזור רנ״ז ולהלן[90].

**Table 14: First Part of the Table: לוח ניגודי המאורות**

| Day | Complete cycles | A.M | Egyptian years | | Days | | min | sec |
|---|---|---|---|---|---|---|---|---|
| שרש | | | | 13m 24s[91] before the epoch which was at noon *of Wednesday 29 Elul 4864 = Wednesday 21 September 1104* | | | | |
| Wednesday | 256 | 4864 | 0 | 0 | 14 | 18 | 8 | 37 |
| Friday | 257 | 4883 | 19 | 0 | 19 | 10 | 41 | 40 |
| Monday | 258 | 4902 | 38 | 0 | 24 | 3 | 14 | 44 |
| Thursday | 259 | 4921 | 57 | 0 | 28 | 19 | 47 | 47 |
| Saturday | 260 | 4940 | 76 | 1 | 3 | 12 | 20 | 50 |
| Tuesday | 261 | 4959 | 95 | 1 | 8 | 4 | 53 | 54 |
| Friday | 262 | 4978 | 114 | 1 | 12 | 21 | 27 | 27 |

90 MS Malatestiana, p. 76a, MS Paris 1046, folio 45a, and Millás Vallicrosa, p. 128.

91 And not 13m 200hal as incorrectly written by Millás Vallicrosa.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sunday | 263 | 4997 | 133 | 1 | 17 | 14 | 0 | 0 |
| Wednesday | 264 | 5016 | 152 | 1 | 22 | 6 | 33 | 3 |
| Saturday | 265 | 5035 | 171 | 1 | 20 | 23 | 7 | 7 |
| Monday | 266 | 5054 | 190 | 2 | 1 | 15 | 39 | 10 |
| Thursday | 267 | 5073 | 209 | 2 | 6 | 8 | 12 | 13 |
| Sunday | 268 | 5092 | 228 | 2 | 11 | 0 | 45 | 17 |
| Tuesday | 269 | 5111 | 247 | 2 | 15 | 17 | 18 | 20 |
| Friday | 270 | 5130 | 266 | 2 | 20 | 9 | 11 | 24 |
| Monday | 271 | 5149 | 285 | 2 | 25 | 2 | 24 | 27 |
| Thursday | 272 | 5168 | 304 | 2 | 29 | 18 | 47 | 30 |
| Saturday | 273 | 5187 | 323 | 3 | 4 | 11 | 30 | 34 [92] |
| Tuesday | 274 | 5206 | 342 | 3 | 9 | 4 | 3 | 37 |
| Friday | 275 | 5225 | 361 | 3 | 13 | 20 | 36 | 40 |

In the present table, the text in italics was added in order to improve the understanding. The transcription by Millás Vallicrosa was mistaken. In contrast with the title, we find on each row the situation at mid-Tishri, after the end of that cycle or at the beginning of next cycle. In contrast with the title, we find on the first row the indications about the first opposition in mid-Tishri 4865, the beginning of the cycle 257. It occurred at 0y 14d 18h 8m 37s after the epoch.

In order to understand and justify Tables 14 and 15, we construct Table 16 on the same basis as Table 12 in order to determine the increment of the parameters in half a lunation. This table is entitled: לוח זה מניגודי מעמד המאורות במחצית תשרי שבראש כל מחזור ממחצית חדש תשרי בראש מחזור רנ"ז ולהלן.[93]

**Table 15: Second Part of the Table: לוח ניגודי המאורות**

| Completed cycles | Common position of sun and moon | | | Moon's anomaly | | | Longitude of moon's ascending node | | |
|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | deg | min | sec | deg | min | sec |
| | שרש: *13m 24s before noon of Wednesday 21 September 1104* | | | | | | | | |
| 256 | 201 | 32 | 38 | 184 | 4 | 49 | 200[94] | 17 | 56 |
| 257 | 201 | 32 | 49 | 131 | 0 | 22 | 207 | 53 | 9 |
| 258 | 201 | 32 | 59 | 77 | 55 | 55 | 215 | 28 | 22 |

92 Instead of 24 in MS Berlin.

93 MS Maletestiana, p. 76a, MS Paris 1046, folio 45a, and Millás Vallicrosa (1959), p. 129.

94 200 according to MS Berlin and MS Paris, instead of 4 as in MS Maletestiana, p. 76a, and in the table printed by Millás Vallicrosa (1959), p. 129.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 259 | 201 | 33[95] | 10 | 24 | 51 | 29 | 223[96] | 3 | 35 |
| 260 | 201 | 33[97] | 20 | 331 | 47 | 2 | 230 | 38 | 48 |
| 261 | 201 | 33 | 31 | 278 | 42 | 35 | 238 | 14 | 0 |
| 262 | 201 | 33 | 41 | 225 | 38 | 8 | 245 | 49 | 13 |
| 263 | 201 | 33 | 52 | 172 | 33 | 42 | 253 | 24 | 26 |
| 264 | 201 | 34 | 2[98] | 119 | 29 | 15 | 260 | 59 | 39 |
| 265 | 201 | 34 | 13 | 66 | 24 | 48 | 268[99] | 34 | 52 |
| 266 | 201 | 34 | 23 | 13 | 20 | 21 | 276 | 10 | 5 |
| 267 | 201 | 34 | 34 | 320 | 15 | 55 | 283 | 45 | 18 |
| 268 | 201 | 34 | 44 | 267 | 11 | 28 | 291 | 20 | 30 |
| 269 | 201 | 34 | 55 | 214 | 7 | 1 | 298 | 55 | 43 |
| 270 | 201 | 35 | 5 | 161 | 20 | 34 | 306 | 30 | 57 |
| 271 | 201 | 35 | 16 | 107 | 58 | 8 | 314 | 6 | 9 |
| 272 | 201 | 35 | 26 | 54 | 53 | 41 | 321 | 41 | 22 |
| 273 | 201 | 35 | 36 | 1 | 49 | 14 | 329 | 16 | 35 |
| 274 | 201 | 35 | 47 | 308 | 44 | 48 | 336[100] | 51 | 48 |
| 275 | 201 | 35 | 57 | 62 | 45 | 51 | 344 | 27 | 0 |

In the present table, the text in italics was added in order to improve the understanding. The transcription by Millás Vallicrosa, p. 129, was seriously flawed by many misprints. In contrast to the title, we find on each row the situation at mid-Tishri following the end of that cycle or at the beginning of next cycle.

Similarly, we ascertain that the span of time indicated on the first row of Table 14 is exactly the difference between 14d 18h 22m 1.67s, half of the length of a lunation and 13m 24s, i.e. 14d 18h 8m 37.67s. This represents the span of time between the epoch and the opposition of Tishri 4865, the first year of the cycle

95 According to MS Malatestiana, p. 76a, and MS Berlin and Paris, instead of 32 in Millás Vallicrosa (1959), p. 129.

96 According to MS Malatestiana and MS Paris, and not 228 according to MS Berlin and Millás Vallicrosa (1959), p. 129.

97 According to MS Malatestiana, p. 76a, and MS Berlin and Paris, instead of 32 in Millás Vallicrosa (1959), p. 129.

98 According to MS Malatestiana, p. 76a, MS Paris and MS Berlin, instead of 20 in Millás Vallicrosa (1959), p. 129.

99 According to MS Malatestiana, p. 76a and not 267 according to MS Berlin, Paris, and Millás Vallicrosa (1959), p. 129.

100 According to MS Malatestiana, p. 76a and not 337 according to MS Berlin, Paris, and Millás Vallicrosa (1959), p. 129.

257. The rows of Table 14 are then deduced from the former row by the addition of 14d 18h 22m 1.67s, and the rows of Table 15 are deduced from the former by the addition of the increment calculated in Table 12.

We used the results of Table 16 to debug misprints. We noted also some contradictions in the last digit of some numbers and a difference of one unit. It is much more difficult to take a position in such cases. Indeed, the last figures are rounded off and we do not know when, in the original calculations, the jump of one unit occurred, on this row or on the next.

### Increment of the Astronomical Parameters in Half a Month

**Table 16. Calculation According to the Tables of Ptolemy of the Increment of the Four Parameters of Sun and Moon During a Period of a Half Lunation or 14d 18h 396.5 hal**

| | Sun's mean longitude | | | | Moon's mean longitude | | | | Moon's anomaly | | | | Argument of latitude | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | deg | min | sec | III | deg | min | sec | III | deg | min | sec | III | deg | min | sec | III |
| 14d | 13 | 47 | 56 | 1 | 184 | 28 | 9 | 40 | 182 | 54 | 35 | 8 | 185 | 12 | 39 | 17 |
| 18h | 0 | 44 | 21 | 12 | 9 | 52 | 56 | 14 | 9 | 47 | 55 | 27 | 9 | 55 | 19 | 15 |
| 396.5 | 0 | 0 | 54 | 20 | 0 | 12 | 5 | 54 | 0 | 11 | 59 | 34 | 0 | 12 | 8 | 23 |
| total | 14 | 33 | 11 | 33 | 194 | 33 | 11 | 48 | 192 | 54 | 30 | 9 | 195 | 20 | 6 | 55 |

The moon's mean longitude increases by 194; 33, 11, 33°, and the argument of latitude increases by 195; 20, 6,55°. The ascending node shifted backward by –0; 46, 5, 10°.

The earlier Table 12 gives us the increment of the astronomical parameters after a cycle of 19 Jewish years or 235 lunations. This gives us a justification of the different numbers of this table. It allows us also to debug the different misprints that abound in these tables in the different manuscripts.

### The Table of the Conjunction and Increment of Astronomical Parameters During a Cycle of 12 Jewish Months

לוח תותרת חדשים לחבור ולנגוד.[101]

This table is practically identical to the inferior table of Toomer, p. 280. It gives the span of time, the common sun and moon longitude, the moon's anomaly, and the moon's argument of latitude at the end of each Jewish month. The only difference is that the left-hand column is not the argument of latitude but the longitude of the

101 MS Malatestiana, p. 74a, MS Paris 1046, folio 44a.

ascending node.

For example, after a month, Ptolemy's table gives a longitude of 29; 6, 23° and 30; 40, 14° for the argument of latitude. We deduce the longitude of the ascending node: 29; 6, 23°– 30; 40, 14° = – 1; 33, 51°. The negative sign is omitted in the tables, but it is well known that this number is always negative.

Besides, at the end of each month the table also gives the span of time between the beginning of the first month and the end of the current month in days, hours, minutes, and seconds.

## The Table of the Conjunction and Increment of Astronomical Parameters During a Cycle of 19 Jewish Years

לוח תותרת השנים בחבורים ובנגודים.[102]

**Table 17: Considering a Cycle of 19 Jewish Years or 235 Lunations**

| days | Span of time between the *molad* at the beginning of the cycle and the *molad* at the beginning of each year in Egyptian years | | | | | | | | Sun's and moon's longitude | | | Moon's anomaly | | | Ascending node | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | c | y | M | d | h | m | s | ''' | deg | m | s | deg | m | s | deg | m | s |
| Wednesday | 1 | 0 | 11 | 24 | 8 | 48 | 40 | | 349 | 16 | 36 | 309 | 48 | 2 | 18 | 46 | 13 |
| Sunday | 2 | 1 | 11 | 13 | 17 | 37 | 20 | | 338 | 33 | 12 | 59 | 36 | 4 | 37 | 32 | 27 |
| Saturday | 3 | 2 | 12 | 2 | 15 | 10 | 3 | 20 | 357 | 56 | 11 | 235 | 13 | 5 | 57 | 52 | 31 |
| Wednesday | 4 | 3 | 11 | 21 | 23 | 58 | 43 | 20 | 346 | 12 | 46 | 185 | 1 | 7 | 76 | 38 | 44 |
| Monday | 5 | 4 | 11 | 11 | 8 | 47 | 23 | 20 | 335 | 29 | 22 | 134 | 49 | 0 | 95 | 24 | 58 |
| Sunday | 6 | 5 | 12 | 0 | 6 | 20 | 6 | 40 | 353 | 52 | 23 | 110 | 26 | 10 | 115 | 45 | 2 |
| Thursday | 7 | 6 | 11 | 19 | 15 | 8 | 46 | 40 | 343 | 8 | 59 | 60 | 14 | 12 | 134 | 31 | 16 |
| Wednesday | 8 | 8 | 0 | 3 | 12 | 41 | 30 | | 1 | 31 | 59 | 38 | 51 | 14 | 154 | 51 | 21 |
| Sunday | 9 | 8 | 11 | 27 | 21 | 30 | 10 | | 350 | 39 | 16 | 345 | 39 | 16 | 173 | 37 | 34 |
| Friday | 10 | 9 | 11 | 17 | 6 | 18 | 50 | | 340 | 5 | 11 | 295 | 27 | 18 | 192 | 23 | 48 |
| Thursday | 11 | 11 | 0 | 1 | 3 | 51 | 33 | 20 | 358 | 28 | 11 | 275 | 4 | 19 | 212 | 43 | 23 |
| Monday | 12 | 11 | 11 | 25 | 12 | 40 | 13 | 20 | 347 | 44 | 47 | 220 | 52 | 21 | 231 | 30 | 5 |
| Friday | 13 | 12 | 11 | 14 | 21 | 28 | 53 | 20 | 337 | 1 | 23 | 170 | 40 | 23 | 250 | 16 | 18 |
| Thursday | 14 | 13 | 12 | 3 | 19 | 1 | 36 | 40 | 355 | 24 | 23 | 146 | 17 | 24 | 270 | 36 | 24 |
| Tuesday | 15 | 14 | 11 | 23 | 3 | 50 | 16 | 40 | 344 | 40 | 59 | 96 | 5 | 26 | 289 | 22 | 38 |
| Saturday | 16 | 15 | 11 | 12 | 12 | 38 | 56 | 40 | 333 | 57 | 35 | 45 | 53 | 28 | 308 | 8 | 51 |
| Friday | 17 | 16 | 12 | 1 | 10 | 11 | 40 | | 352 | 2 | 35 | 21 | 30 | 29 | 328 | 28 | 56 |
| Tuesday | 18 | 17 | 11 | 20 | 19 | 0 | 20 | | 341 | 37 | 11 | 331 | 18 | 31 | 347 | 15 | 9 |
| Monday | 19 | 19 | 0 | 4 | 16 | 33 | 3 | 20 | 0 | 0 | 10 | 306 | 55 | 33 | 7 | 35 | 13 |

102 MS Malatestiana, p. 75, MS Paris 1046, folio 44a, and Millás Vallicrosa, p. 123.

The present table gives the span of time between the first *molad* at the beginning of the cycle and the *molad* of each Jewish year, counted in Egyptian years and the astronomic parameters of sun and moon at the end of each cycle of 19 Jewish years or at the beginning of next cycle. In the left-hand column, sub-column c is the rank of the considered year in the 19-year cycle, y is the number of elapsed Egyptian years, M the number of additional months of 30 days, d h, m and s the additional days, hours, minutes, and seconds. The three last columns correspond to the evolution of the astronomical parameters after 12, 24, 37, 49, 61, 74, 86, 99, 111, 123, 136, 148, 160, 173, 185, 197, 210, 222, and 235 months. In the left-hand column, giving the span of time until the *molad* of the beginning of each year, the underscored figures differ from the figures of the table of Millás Vallicrosa, p. 123, of *Sefer Heshbon Mehalekhot ha-Kokhavim*, and MS Berlin. The two other MS present numerous misprints. Therefore, the table must be completely recalculated.

## 9. TABLES ACCORDING TO AL-BATTĀNĪ

We do not generally[103] find tables calculated according to al-Battānī in Abraham bar Hiyya's book. Toward the end of the book, however, we find several tables allowing the calculation of the corrections to add to or subtract from the main tables according to Ptolemy, in order to obtain the values of the corresponding astronomical sizes according to al-Battānī.

### 9a. Mean Motion of the Sun and the Moon

The table is entitled:

תותרת מהלך חמה ולבנה השוה לדעת אלבתאני על מהלך השוה לדעת בטלמיוס.[104]

**Motion of the Sun**

Ptolemy: tropical year 365d 5h 55m 12s = 365.246666666d and $\omega_{sol}$ = 0.985635278441 °/d

al-Battānī: tropical year 365d 5h 46m24s = 365.24055555d and $\omega_{sol}$ = 0.985651769837 °/d

The difference is thus 0.000016491396 °/d.

In 30 cycles of 19 Egyptian years the difference of motion is 3.4310° = 3; 25, 52°.

The table indicates for 30 cycles 3; 25’, 52° in MS Paris but 3; 25, 55° in MS Malatestiana.

103 There are a few exceptions, like the three tables of the quota of the anomaly of the sun and the moon, and the movement of the apogee of the sun and the planets.

104 MS Malatestiana, p. 67b, and MS Paris 1046, folio 39a.

**Motion of the Moon**

Ptolemy: The mean motion in a day is 13; 10, 34.97597 °/d.
al-Battānī: The mean motion in a day is 13; 10, 35.03534 °/d.
Difference: 0.05937''/d
In 30 cycles of 19 Egyptian years the difference of motion is 12351.93'' = 3; 25, 51.93°.
The additional correction is thus practically the same for the sun and the moon,[105] and the precision of Abraham bar Hiyya's calculation is remarkable.

### 9b. Motion of the Ascending Node of the Moon

חסרון מהלך ראש התלי וזנבו לדעת אלבתאני מן מהלכו לדעת בטלמיוס.[106]

From the tables of al-Battānī, Nallino (1903–05), Vol. 2, p. 77, we find the following data: movement in 600 Roman years = 219,150 days:
Increment in moon's longitude: 47; 46, 36°
Increment in ascending node longitude: – 84; 11, 41°.

From Ptolemy's tables, Toomer, pp. 182–87, we find, taking into account that 600 Roman years = 594 y + 6 y + 150d = 600 * 365 + 150 = 219,150 days, where y is an Egyptian year of 365 days.

| | Lunar mean longitude | Argument of latitude |
|---|---|---|
| 594y | 171; 25, 41, 2° | 135; 35, 24, 8° |
| 6y | 56; 16, 37, 23° | 172; 16, 43, 16° |
| 150d | 176; 27, 26, 23° | 184; 24, 9, 32° |
| | 44; 9, 44, 48° | 132; 16, 16, 56° |
| | | – 44; 9, 44, 48° |
| | | 88; 6, 32, 8° |

**Motion of the Moon**

In 600 Roman years or 219,150 days: movement al-Battānī – movement Ptolemy is: (47; 46, 36°) – (44; 9, 45°) = 3; 36, 51°. After 30 cycles of 19 Egyptian years or 208,050 days the difference is 3; 25, 52°. This is the result already found above on the same page.

105 Therefore, they were presented in one unique table.
106 MS Malatestiana, p. 68a, and MS Paris, folio 40a.

**Motion of the Ascending Node**

In 600 Roman years or 219,150 days: motion Ptolemy – motion al-Battānī, in absolute value, is: (88; 6, 32°) – (84; 11, 41°) = 3; 54, 51°. After 30 cycles of 19 Egyptian years or 208,050 days the difference is 3; 42, 57°, as compared with 3; 19, 30° given in the table of Abraham bar Hiyya.

## 9c. Motion of the Planets

**Introduction**

Abraham bar Hiyya tabulated the motions of the planets according to Ptolemy, but according to the principles adopted by al-Battānī. Indeed, Ptolemy tabulated separately, and very easily, the mean motion in longitude and in anomaly of each of the five planets, thus in fact ten tables. Abraham bar Hiyya, like al-Battānī, but without any explanation, limited himself to five tables, the longitude of the three superior planets, and the anomaly of the two inferior planets. This is the result of the following properties, which we ascertain in the tables of Ptolemy.

For the superior planets, the sum of the motion in longitude and in anomaly is equal to the motion of the sun during the same period.

For the inferior planets, the mean motion in longitude is equal to the mean motion in longitude of the sun.

Ptolemy gives the following radices for Saturn, position: 296; 43°; apogee: 224; 10° and 330; 45° for the position of the sun. Hence, the radix of the anomaly is: 34; 2°. We check that the position of Saturn = position sun – anomaly Saturn and 296; 43° = 330; 45° – 34; 2°.

**Saturn**[107]

**מהלך השוה לדעת אלבתאני על מהלכו לדעת בטלמיוס של שבתאי תותרת**

We compare the movement of Saturn in 600 Roman years between Ptolemy and al-Battānī. Al-Battānī, Vol. 2, p. 103: The movement is 141; 23°.

According to Ptolemy, taking into account that 600 Roman years = 208050 days = 600 Egyptian years + 150 days.

| | | |
|---|---|---|
| Toomer, p. 427 | 594 y | 60; 39, 1, 25, 59° |
| Toomer, p. 428 | 6 y | 73; 20, 23, 39, 3° |
| Toomer, p. 429 | 150 d | 5; 1, 23, 48, 42° |

107 MS Malatestiana, p. 68b, and MS Paris, folio 40a.

| | |
|---|---|
| 600 Roman years: | 139; 0, 48, 53, 44° |
| al-Battānī | 141; 23° |
| Difference | 2; 22° for 219150 days. |

In 30*19*365 = 208050d we find a difference of: 2; 14, 48, 36°.

Abraham bar Hiyya gives for the last entry of 30 cycles of 19 Egyptian years 2; 15, 9, 45° in MS Malatestiana, but 2; 15, 36, 45° in MS Paris.

**Jupiter**[108]

תותרת מהלך השוה לדעת אלבתאני על מהלכו לדעת בטלמיוס של צדק.

| | | |
|---|---|---|
| Toomer, p. 430 | 594 y | 21; 46, 31, 32° |
| Toomer, p. 431 | 6 y | 182; 2, 17, 18° |
| Toomer, p. 432 | 150 d | 12; 28, 6, 7° |
| 600 Roman years: | | 216; 16, 54, 57° |
| al-Battānī | | 218; 47° |
| Difference | | 2; 30° for 219150 days. |

In 30*19*365 = 208050d we find a difference of: 2; 22°.

Abraham bar Hiyya gives for the last entry of 30 cycles of 19 Egyptian years 2; 20, 59, 17°.

**Mars**[109]

תותרת מהלך השוה לדעת אלבתאני על מהלכו לדעת בטלמיוס של מאדים.

| | | |
|---|---|---|
| Toomer, p. 433 | 594 y | 221; 23, 9, 40° |
| Toomer, p. 434 | 6 y | 67; 41, 26, 46° |
| Toomer, p. 435 | 150 d | 78; 36, 32, 15° |
| 600 Roman years: | | 7; 41, 8, 41° |
| al-Battānī | | 11; 5° |
| Difference | | 3; 24° for 219150 days. |

In 30*19*365 = 208050d we find a difference of: 3; 13.67°.

Abraham bar Hiyya gives for the last entry of 30 cycles of 19 Egyptian years 3; 20, 16, 5°.

108 MS Malatestiana, p. 69a, and MS Paris, folio 40b.
109 MS Malatestiana, p. 69b, and MS Paris, folio 40b.

**Anomaly of Venus**[110]

.תותרת מהלך החוק בנגה לדעת אלבתאני על מהלכו לדעת בטלמיוס

| | | |
|---|---|---|
| Toomer, p. 436 | 594 y | 105: 15, 30, 55° |
| Toomer, p. 437 | 6 y | 270; 9, 14, 51° |
| Toomer, p. 438 | 150 d | 92; 28, 34, 43° |
| 600 Roman years: | | 107; 53, 20, 29° |
| al-Battānī | | 111; 31° |
| Difference | | 3; 38° for 219150 days. |

In 30*19*365 = 208050d we find a difference of: 3; 26.96°.

Abraham bar Hiyya gives for the last entry of 30 cycles of 19 Egyptian years 3; 24, 27, 16°.

**Anomaly of Mercury**[111]

.תותרת מהלך החק בכותב לדעת אלבתאני על מהלכו לדעת בטלמיוס

| | | |
|---|---|---|
| Toomer, p. 439 | 594 y | 3: 25, 10, 15° |
| Toomer, p. 440 | 6 y | 323; 20, 23, 39° |
| Toomer, p. 441 | 150 d | 106; 0, 17, 29° |
| 600 Roman years: | | 73; 5, 42, 59° |
| al-Battānī | | 73; 52° |
| Difference | | 0; 46° for 219150 days. |

In 30*19*365 = 208050d we find a difference of: 0; 44°.

Abraham bar Hiyya gives for the last entry of 30 cycles of 19 Egyptian years 1; 32, 28°.

The discordance between the data of the tables for the difference al-Battānī – Ptolemy and our calculations is surprising in the case of Mercury. Our data are nevertheless confirmed by Nallino in Vol. 1, p. 242, where he calculated the difference of the motion of the anomaly of Venus and Mercury in 740 Egyptian years and found for Venus a difference of 4; 28, 26, 44°, and for Mercury a difference of 0; 56, 40, 20°. These differences correspond to a span of time of 270,100 days. For a span of time of 219,150 days or 600 Roman years, we get 3; 37, 48° for Venus and 0; 45, 59° for Mercury, in perfect concordance with our calculations.

110 MS Malatestiana, p. 70a, and MS Paris, folio 41a.
111 MS Malatestiana, p. 70b, and MS Paris, folio 41a.

## Astronomical Positions According to al-Battānī

The former tables allow us to calculate the motions of the celestial bodies according to al-Battānī. Nevertheless, we need the radices according to al-Battānī in order to calculate the mean positions at any moment.

### Radices According to al-Battānī

We find important indications in a text following these tables and belonging to the second part of the book *Sefer Heshbon Mehalekhot ha-Kokhavim*.[112] We read the following data at the beginning, at the epoch of the beginning of the cycle 257. All the following figures must be subtracted from the "improved" radices of the tables calculated according to Ptolemy.

Saturn: 0; 0, 4°. Jupiter: 0; 0, 12°. Mars: 0; 1, 22°. Sun: 0; 2, 36°.

Venus: 0; 1, 36°. Mercury: 0; 8, 9°. Ascending node: 0; 2, 39°.

If we refer to Table 8, we note the perfect correspondence between the indications of the manuscript and the result of our calculations. However, we note discordance for Mars and for the ascending node of the moon, which must be the result of scribal error.

### Apogee

We find data about the apogees according to Ptolemy in the first part of the book *Sefer Heshbon Mehalekhot ha-Kokhavim*, which constitutes the canon of the tables. We find data about the same apogees according to al-Battānī in a text belonging to the tables.[113]

112 MS Maletestiana, pp. 66a-67a. The text is practically unreadable and deficient. MS Berlin is more helpful.

113 MS Malatestiana, p. 67a.

**Table 18: The Apogees of the Sun and the Planets**

| Apogee of the sun and the planets | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Ptolemy | | al-Battānī | | Abraham bar Hiyya | | |
| | Epoch of Nabonassar | 21 Sept 1104 | March 880 | 21 Sept 1104 | Vol. 1, p. 70 according Ptolemy[114] | Tables al-Battani Malatest. | Tables al-Battānī Berlin[115] |
| Sun | 65; 30° | 75;30° | 82;14° | 85;38° | 75;30° | 85; 40° | 85;40° |
| Saturn | 164;10° | 182;41° | 244;28° | 247;52° | 243;53,12° | 247; 22° | 247;52° |
| Jupiter | 152; 9° | 170;40° | 164;28° | 167;52° | 172;22, 8° | 175° | 175;52° |
| Mars | 106;40° | 125;11° | 126;58° | 130;22° | 126;21,52° | 130; 42° | 130;42° |
| Venus | 46;10° | 64;41° | 82;14° | 85;38° | 75;30° | 85; 40° | 85;40° |
| Mercury | 181;10° | 199;41° | 201;28° | 204;52° | 201° | 204; 52° | 204;52° |

This table presents all the available elements.

This table requires some explanations. According to Ptolemy, the apogees are fixed on the eighth sphere, the sphere of the fixed stars. It has a slow direct movement of 1° in 100 Egyptian years. Al-Battānī has a similar conception, shared also by Maimonides,[116] but the movement of the eighth sphere is 1° in 66 years. In the Almagest, the apogee of the sun, strangely, is always 65; 30°. It seems that the precession of the equinox does not concern the apogee of the sun.

Abraham bar Hiyya does not share this position. He assumes that Ptolemy measured the position of the apogee at 65; 30°, and he adds to this figure the precession from the time of Ptolemy (about 137) until his epoch (21 September 1104), about 9°; 40’, which he rounds off to 10°.

For the other planets he uses the apogees given by Ptolemy at the head of his planetary tables in the epoch of Nabonassar (Toomer [1984], pp. 427-41), and he adds a precession of 1° in 100 Egyptian years for a span of time of 1104 – (– 746) = 1850 years, i.e. 18.5°.

Al-Battānī gave the longitude of the apogees at the head of his planetary tables[117] for the year 880, and we calculated the value in the epoch of Abraham bar Hiyya

114 See first part, pp. 70-71. Abraham bar Hiyya wrote (bottom p. 70) the enigmatic text: גובה הרום בראש מחזור רנ"ז לפי הנראה לנו בחשבון בטלמיוס שהזכיר בספרו.

115 And MS Paris. The figures are mentioned at the bottom of folio 39a of MS Paris, but they are difficult to read.

116 Rambam, Hilkhot Yessodei ha-Torah; 3:7. He speaks of 1° in 70 years.

117 Pp. 108, 114, 120, 126, and 132.

by taking into account a precession of about 3°; 24'.[118] We note the quasi-perfect coincidence between this column and the column with the data of MSS Berlin and Paris. However, the apogee of Jupiter raises a problem and scribal error is likely. The column of the apogees according to Ptolemy, mentioned in the text of the first part of the book,[119] also raises many problems.

Apparently, Abraham bar Hiyya realized that the values of Ptolemy were not acceptable and tried to adapt them. In contrast to the other parameters of the table, we do not see clearly how he proceeded. We may even suspect scribal error, because the value adopted for the apogee of Saturn seems exceptionally high. For the other planets, the difference is limited, and one does not in fact understand why he changed the value of Ptolemy at all.

**Table 19: Recapitulative Table of the Radices of the Planets on 21 September 1104 at Noon According to al-Battānī**

| al-Battānī: Radices of the planets on Wednesday 21 September 1104 at noon | | | | | |
|---|---|---|---|---|---|
| Radix | Saturn | Jupiter | Mars | Venus | Mercury |
| Longitude | 256; 8° | 213; 36° | 351; 53° | 186: 57, 24° | 186; 57, 24° |
| Anomaly | 290; 49, 24° | 333; 21, 24° | 195; 4, 24° | 267; 33° | 25; 46° |
| Apogee | 247; 52° | 167; 52° | 130; 22° | 85; 38° | 204; 52° |

## 10 MISCELLANEOUS: OTHER ASTRONOMICAL TABLES

### 10a. Table of Sines

This table is entitled: לוח הקשתות והמיתרים המחצים[120]. The table certainly is based on that of al-Battānī. However, al-Battānī's table gives the sinus of the angles in degrees and half degrees, while that of Abraham bar Hiyya gives only the sinus of the angles in degrees (integers). Al-Battānī's editor complained already about the number of misprints in the former texts. The same can be said here. The sinus is given in sexagesimal notation.[121] For example, instead of sin 30 = 0.5 we find sin

118 1° in 66 years applied on a span of time of 224.56 Roman years.
119 P. 70 of the printed text.
120 MS Malatestiana, p. 8a, and MS Paris, folio 7a and b, 53 a and b. The sinus is thus called the half of the chord of the half angle and it refers directly to Ptolemy's table of the chords, Toomer (1984), pp. 57-59.
121 Until the end of the sixteenth century, the trigonometric functions were calculated and tabulated according to the sexagesimal notation. Viete (1579) urged the use of decimal rather than sexagesimal notation because of its advantage. Delmedigo (1629) produced

30 = 30p; 0’, 0’’. Moreover, sin 24 = 0.406736 becomes sin 24 = 24p; 24’, 15’’.

### 10b. Table of Right Ascension

The title of this table is: מצעדי המזלות על מפריש קו היושר ועל קשת חצי השמים[122] .

### Introduction

Any plane of horizon of a point of the equator is parallel to the axis of rotation of the earth. Furthermore, the equator and the parallels are perpendicular to this horizon. The setting and rising of the sun and stars are perpendicular to the horizon. The circle of declination passing through the considered point of the ecliptic contains the axis of rotation of the earth, and it is parallel to the plane of the horizon of a certain point of the terrestrial equator. Therefore, the ancients would say that the considered point of the ecliptic and the point of the celestial equator situated on the same circle of declination rise and set together on the right horizon, i.e. the horizon of a point of the equator and also on the meridian. Indeed, during the diurnal rotation, the circle of declination coincides, twice a day, with the meridian. The particular configuration at the equator, with rising and setting curves perpendicular to the horizon was called “sphaera recta,” and it was said that the two points on the same circle of declination rise and set together on the right horizon or on “sphaera recta,” and the denomination “right ascension” recalls this ancient conception.

The table of the right ascension of Abraham bar Hiyya is constructed on the same basis as that of al-Battānī,[123] and, for an unknown reason, presents the strange particularity that the arcs of the equator begin at the beginning of Capricornus (270°) instead of at the beginning of Aries (0°), in this table. Therefore, the angles of right ascension given in the two tables are the right ascension + 90°. In other words, an angle of 90° must be subtracted from the values given in both tables. We note that the data of the two tables, that of Bar Hiyya and that of al-Battānī,

the first printed table of sines and cosines in Hebrew. It was established in decimal notation. See Loewinger (2006), for a paper about the evolution of this table in history.

Already in the first half of the 12th century, a table of sines in Hebrew circulated among educated Jewish individuals. It is also by chance that the table of Delmedigo was published in decimal notation. Without the publication of François Viete, the table of *Sefer Elim* would have been published in sexagesimal notation.

122 MS Malatestiana, p. 11a, and MS Paris, folio 8b and 9a.

123 Vol. 2, pp. 63-64.

are systematically slightly divergent.

For example, for $\lambda = 30°$, al-Battānī writes 117; 53° – 90° = 27; 53° while bar Hiyya gives 27; 50°, in fact the same value as Ptolemy.[124] It is easy to check with the formula: $\tan \alpha = \cos \varepsilon * \tan \lambda$, that the differences between the tables of bar Hiyya and al-Battānī result from the different value adopted for $\varepsilon$: al-Battānī uses 23; 35°, as will be adopted by all future astronomers. Abraham bar Hiyya recalculated his tables with the old value of Ptolemy, $\varepsilon$ = 23; 51, 20°. The use of this value in the mid-12th century was almost anachronistic.

## 10c. Table of Oblique Rising

**Introduction**

The hour angle of the sun, when it rises or sets at the horizon, is given by the formula: $\cos H = -\tan \varphi * \tan \delta$, where $\varphi$ is the geographical latitude of the considered place and $\delta$ the declination of the sun.

The sidereal time is the hour angle of the vernal point. It is measured from the superior point of the celestial equator, which is also on the superior meridian. It is positive from 0° to 180° toward the west, and negative from 0° to 180° toward the east. We have the identity $Ts = H + \alpha$, where H is positive at sunset and negative at sunrise. In spring and summer, $\delta$ is positive, cos H is negative, and $H = 90° + \Delta > 90°$ and $\Delta > 0$. In autumn and winter, $\delta$ is negative, cos H is positive, and $H = 90° + \Delta < 90°$ and $\Delta < 0$.

$\sin \Delta = \sin (H - 90°) = -\cos H = \tan \varphi * \tan \delta$.

At sunset: $Ts = \alpha + H = \alpha + 90° + \Delta = 90° + (\alpha + \Delta)$.
At sunrise: $Ts = \alpha - H = \alpha - (90° + \Delta) = (\alpha - \Delta) - 90° = -[90° - (\alpha - \Delta)]$.

The ancients did not use sidereal time, and did not refer to the superior point of the celestial equator as a reference point. They simply measured the coordinate of the point of the equator, its right ascension, rising or setting at the oblique horizon, together with the considered point of the ecliptic. The right ascension of the point of the equator setting together with the point $\lambda$ of the ecliptic is $(\alpha + \Delta)$, and that of the point of the equator rising together is $(\alpha - \Delta)$. We speak of an oblique horizon for the horizon of any location that is not on the equator. The equator and parallels are not more perpendicular than the plane of the horizon, and the rising and setting of the sun and stars is then indeed oblique with regard to the horizon.

124 Toomer (1984), p. 100.

In order to demystify the problem let us consider two examples.

**First Example**

Suppose $\lambda = 60°$ (we are the 21st of May), $\alpha = 57.5°$ and $\Delta = 15°$.

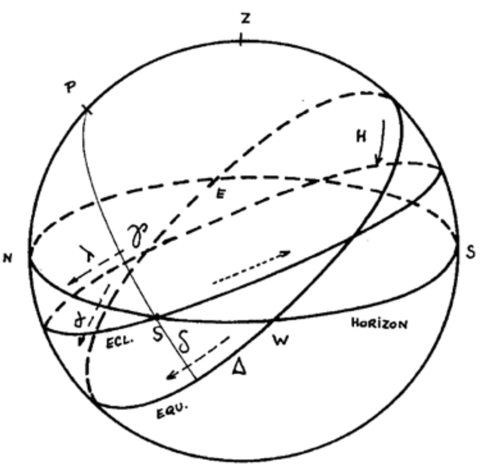
Sunset. The vernal point $\gamma$ is under the western horizon. The hour angle of the setting sun is $90° + \Delta = 105°$. The length of the day is 210° or 14 hours. The sidereal time at sunset is $\alpha + H = 57.5 + 105 = 162.5°$. The right ascension of the intersection of the equator and horizon is $\alpha + \Delta = Ts - 90° = 72.5°$.

**Sunrise**. The vernal point $\gamma$ is above the eastern horizon. The hour angle of the rising sun is $-(90° + \Delta) = -105°$. The sidereal time at sunrise is $\alpha + H = 57.5 - 105 = -47.5°$. The right ascension of the intersection of the equator and horizon is $\alpha - \Delta = 57.5 - 15 = Ts + 90° = -47.5° + 90° = 42.5°$.

**Second Example**

Suppose $\lambda = 240°$ (we are the 21st of November), $\alpha = 237.5°$ and $\Delta = -15°$.

**Figure 3: Representation of Sunset and the Different Astronomical Parameters**




On this 32.46°.

**Sunset**. The vernal point $\gamma$ is above the eastern horizon. The hour angle of the setting sun is $90° + \Delta = 75°$. The length of the day is 150° or 10 hours. The sidereal time at sunset is $\alpha + H = 237.5 + 75 = 312.5°$. The right ascension of the intersection of the equator and horizon is $\alpha + \Delta = 237.5 - 15 = 222.5° = Ts - 90° = 312.5° - 90° = 222.5°$.

**Sunrise.** The vernal point $\gamma$ is under the western horizon. The hour angle of the rising sun is $-(90° + \Delta) = -75°$. The sidereal time at sunrise is $\alpha + H = 237.5 - 75 = 162.5°$. The right ascension of the intersection of the equator and horizon is $\alpha - \Delta = 237.5 + 15 = 252.5° = Ts + 90° = 162.5° + 90° = 252.5° = -107.5°$.

**The Table of Oblique Rising Allows Calculating an Oblique Setting**

Oblique setting (60°) = oblique rising (60° + 180°) – 180° = 252.5° – 180° = 72.5°.

**The Tables of Oblique Rising[125] of Savasorda**

They are entitled, for example:

מצעד המזלות במרחב י״ו ל״ב ושעותיו י״ג[126]

The meaning of this title is as follows: we want to know the point of the equator rising together with a point of the ecliptic for an observer in a location of latitude 16; 32°, where the longest day of the year reaches the length of 13 equinoctial hours. In the table of al-Battānī, the latitude is 16; 39° and the length of the longest day is 13h. Ptolemy gets a longest day of 13h for the latitude of 16; 27°. The origin of the difference between al-Battānī and Savasorda is again the value adopted for $\varepsilon$; 23; 35° for al-Battānī, and 23; 51, 20° for bar Hiyya and Ptolemy. It is also surprising that the tables of Abraham bar Hiyya were given to three sexagesimal places, while the right ascension was given to only one sexagesimal place!

If we consider $\lambda = 50°$, we read the following results for the point of the equator rising together: al-Battānī 42; 3° Ptolemy: 41; 57, 3° and Savasorda 41; 57, 15° MS Malatestiana and 41; 57, 16° MS Paris.

**al-Battānī.** $\varphi = 16; 39°$, $\delta = 17.8473°$ and $\alpha = 47.5238°$.

The longest day: $\cos H = -\text{tang}(16; 39°) * \text{tang}(23; 35°) = -0.13055$. $H = 97.5016°$.

The length of the day is 195.0032°= 13.0002 h.

$\Delta = \text{tang}(16; 39°) * \text{tang}(17.8473) = 0.09629$. $\Delta = 5.5256°$. $\alpha - \Delta = 41.9981 = 41; 59, 53°$.

**Ptolemy**. $\varphi = 16; 27°$, $\delta = 18.0480°$ and $\alpha = 47.4642°$. $\Delta = 7.5024°$.

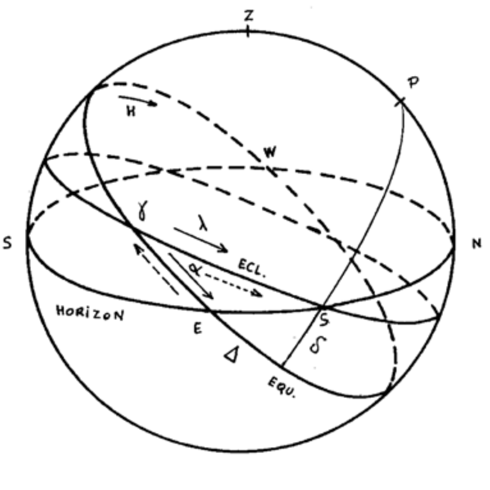
125 The ancients, Ptolemy, al-Battānī, and Abraham bar Hiyya tabulated the oblique rising $\alpha - \Delta$. In Hannover (1756), the author tabulated the oblique setting $\alpha + \Delta$.

126 MS Malatestiana, pp. 14a-18a, and MS Paris, folio 10, 11 and 12a. These tables are very similar to the tables of al-Battānī, pp. 65-68.

The longest day: cos H = – tang (16; 27°) * tang (23; 51, 20°) = –0.13057. H = 97.5025°.
The length of the day is 195.0049° = 13.0003 h. $\alpha - \Delta$ = 39.9618° = 39; 57, 42°
$\Delta$ = tang (16; 27°) * tang(18.0480) = 0.09621. $\Delta$ = 5.5210°. $\alpha - \Delta$ = 41.9432 = 41; 56, 36°.

**bar Hiyya**. $\varphi$ = 16; 32°, $\delta$ = 18.0480° and $\alpha$ = 47.4642°.
The longest day: cos H = – tang (16°; 32') * tang (23; 51, 20°) = – 0.1313. H= 97.5429°.
The length of the day is 195.0858°= 13.0057 h.
$\Delta$ = tang (16; 32°) * tang (18.0480) = 0.09673. $\Delta$ = 5.5507°. $\alpha - \Delta$ = 41.9135 = 41; 54,49°.
The last calculation is slightly less precise than the calculations of Ptolemy and al-Battānī.

**Figure 4: Representation of Sunrise and the Different Astronomical Parameters**




## 10d. Precession of the Equinox According to al-Battānī

מהלך רומי הרום לדעת אלבתאני לששה[127] כוכבי לכת.[128]

The precession is 1° in 66 years. After 330 years, the last entry of the table, the precession is 5°.

## 10e. Equation of the Anomaly of the Sun According to al-Battānī

תקון מהלך החמה לדעת אלבתאני[129]

This table has a structure similar to the first table, but the maximum value of the equation is 1; 59, 10° for an anomaly of 92° and 268°, and is identical to that of al-Battānī.[130]

## 10f. The Shade of a Vertical Gnomon

The title of the table is

צל העומד לפני החמה במעלותיה מן א׳ עד צ׳.[131]

**Table 20: The Shade of a Gnomon, Comparison Between Ptolemy, al-Battānī, and Abraham bar Hiyya**

| Solar altitude in degrees | Abraham bar Hiyya | al-Battānī | Exact decimal calculation | Exact sexadecimal |
|---|---|---|---|---|
| 1 | 687d 26’ | 687d 29’ | 687.479539 | 687d 28’, 46’’ |
| 2 | 343d 39’ | 343d 38’ | 343.635039 | 343d 38’, 6’’ |
| 3 | 228d 58’ | 228d 58’ | 228.973640 | 343d 58’, 25’’ |
| 4 | 171d 34’ | 171d 36’ | 171.607995 | 171d 36’, 29’’ |
| 5 | 137d 4’ | 137d 10’ | 137.160628 | 137d 9’, 38’’ |
| 6 | 104d 10’ | 114d 10’ | 114.172373 | 114d 10’, 21’’ |
| 7 | 94d 44’ | 97d 44’ | 97.732157 | 97d 43’, 56’’ |
| 8 | 85d 23’ | 85d 23’ | 85.384437 | 85d 23’, 4’’ |
| 9 | 75d 46’ | 75d 46’ | 75.765018 | 75d 45’, 54’’ |
| 10 | 68d 3’ | 68d 3’ | 68.055382 | 68d 3’, 19’’ |
| 11 | 61d 44’ | 61d 44’ | 61.734648 | 61d 44’, 5’’ |

The length of the gnomon is 12 and the result is expressed in digits, d = digit.

127 We normally speak of five planets. Perhaps the sixth planet is the sun. The movement of its apogee was identified with the general precession of the equinox.
128 MS Paris, 60a, and MS Maletestiana, p. 28b.
129 MS Malatestiana, pp. 71a and b, and MS Paris, folio 41b-42a.
130 Nallino, Vol. 2, pp. 78-83.
131 MS Malatestiana, p. 10a, and MS Paris, 8b.

The table is identical, apart from misprints and copyist mistakes to that of al-Battānī, Vol. 2, p. 60. The length of the gnomon is assumed to be 12. The length of the shade is 12 * cotg h where h is the altitude of the sun. If h = 30°: 12 cotg 30 = 20.7846 = 20digits 47'; 5''. al-Battānī gives 20d 47'. Bar Hiyya gives the same value. If h = 60°: 12 cotg 60 = 6.9282 = 6d 55' 42''. Bar Hiyya gives 6d 56'.

Remark. Neither Al-Battānī nor bar Hiyya explains how we get the altitude h of the sun at any moment. In fact, we get the altitude of the sun through the formula: Sin h= sin φ * sin δ + cos φ * cos δ * cos H, where H = Ts – α, H and α refer to the sun. H is directly connected to the true time.

### 10g. Table of the Solar Parallax and the Lunar Parallax in Function of the Altitude in the Four Limit Points of the Distance Moon–Earth

לוח חלופי מראות הלבנה באופן המעלות בארבעה גבולי מרחקה[132]

This table presents many similarities with the table of al-Battānī, Vol. 2, pp. 93-94, who refers himself to the Almagest. Indeed, we find a similar table in the Almagest, Toomer (1984), p. 265. We note that the two tables are identical, except in the fifth column, at the third limit, where the figures are slightly different. The figures of our table are similar but not identical.

### 10h. Tables of Lunar Parallax in Longitude and Latitude in the Different Climates[133]

The climates are classified according to the maximum length of the days, and are spaced from 13h until 16h. The title of these tables is for example:

חלוף מראות לבנה באקלים השני במרחב כ״ג נ״ב שעות י״ג ל׳.[134]

The latitudes of the seven climates were recalculated by Abraham bar Hiyya in order to correspond to maximum length of days of 13h, 13.5 h, 14h, and so on until 16h. Indeed, bar Hiyya uses ε = 23; 51, 20° differently from the value measured by al-Battānī. We noted already that the latitudes of his seven climates differ slightly from those of Ptolemy. The presentation and the organization of these tables is the same as the similar tables in al-Battānī. We do not find similar tables in the Almagest.

132 MS Malatestiana, pp. 52, and MS Paris, folio 29a.

133 In ancient astronomy, there are seven climates. Each climate represents a zone of latitude.

134 MS Malatestiana, pp. 53a-55b, and MS Paris, folio 30-34a.

### 10i. Table for the Correction135 of the Longitude of the Planets in Function of the Anomaly[136]

We find similar tables in the Almagest, Toomer (1984), pp. 549-53, and in al-Battānī, Vol. 2, pp. 108-37. We ascertain that the tables of Abraham bar Hiyya and their figures are identical, if we exclude the misprints, to the tables of al-Battānī.

However, we note one difference: in the tables of al-Battānī there are seven columns, while in those of Abraham bar Hiyya there is an eighth column, which I could not, for the moment, explain. It is different from the eighth column in the tables of Ptolemy.

### 10j. Table of the Greatest Elongations with Respect of the True Sun for the Two Inferior Planets

The title of the table in Hebrew is: סוף מרחק נגה וכותב מן חמה למערב ולמזרח[137].
This table is identical to the table of the Almagest, see Toomer (1984), p. 596.

### 10k. Table for the Elongation of the Three Superior Planets, in Order to Know the Apparition and the Occultation in the East and in the West (at the latitude of 36°)

The title of the table in Hebrew is: הגלות שבתאי וצדק ומאדים מאור החמה והסתרם בבאם בתוכה[138]. It corresponds exactly, except for errors and misprints, to the table of al-Battānī, Vol. 2, p. 142.

### 10l.Transformation of Halakim (1080 per Hour) into Minutes (60 per Hour) and Conversely

The tittle of the table in Hebrew is: לוח להשיב חלקי רבותינו שהם תתר״ף לשעה לחלקי חכמי התכונה שהם ס׳ לשעה.
We find in MS Paris 1046 folio 60a one such convenient table. It appears to belong to bar Hiyya's tables. He certainly used this table during the redaction of Tables 10, 14, and 17 above.

135 תקון. In fact, it is what we call today the equation of the center, or the equation of the anomaly.
136 MS Malatestiana, pp. 33b-36a for Saturn, 36b-38b for Jupiter, 39a-42a for Mars, 42b-45a for Venus, and 45b-48a for Mercury. MS Paris, folios 20-26.
137 MS Malatestiana, pp. 58a, and MS Paris, folio 34b.
138 MS Malatestiana, pp. 57a, and MS Paris, folio 34b.

## 10m. Catalogue of Stars

Abraham bar Hiyya gives three tables of stars, the first is devoted to the stars of the first magnitude (14 stars), the second to the main stars of the second magnitude (14 stars),[139] and the third to the other main stars (48 stars).[140] The abridged catalogue of R. Abraham bar Hiyya is constructed for the beginning of the cycle 257, thus for his epoch of 21 September 1104.

The reference catalogue is Ptolemy's monumental catalogue, which contains 1022 + 3 = 1025 stars established for the date of 20 July 137 CE.[141] These 1025 stars are divided into three categories: 360 stars belong to boreal constellations, 349 stars belong to zodiacal constellations, and 316 stars belong to austral constellations. According to al-Battānī, Ptolemy's catalogue was based on a catalogue previously established by Menelaus[142] in 92 CE, which Ptolemy adapted and expanded.[143] Although this attribution is contested, it played an important role in the interpretation and quantification of the precession.[144] Al-Battānī established a comparable catalogue for the year 880 CE.[145]

In order to understand the abridged catalogue of R. Abraham bar Hiyya, we will compare his list of stars of the first magnitude with the lists of the stars of the first magnitude that we extracted from the complete catalogues of stars of Ptolemy and al-Battānī. It is likely, a priori, that the list of the stars deduced from the Almagest is sufficient, but al-Battānī's list is necessary because Abraham bar Hiyya uses Hebrew and Arabic denominations. Al-Battānī's list, which refers to both the description of the localization of the stars like Ptolemy and their Arabic designations, will enable us to compare with the list of Ptolemy.

139 The last star of this second list is "khsil which is Sahil which is of the first magnitude."
140 MS Malatestiana, pp. 58b-59b, and MS Paris, folio 35a-36a.
141 See Toomer, pp. 341-99.
142 Astronomer who lived in Rome in the second half of the first century.
143 See Nallino, *al-Battānī Opus Astronomicum* (Milano, 1903), Vol. 1, p. 124 and p. 292.
144 See Nallino, Vol. 2, pp. 269-70, and Zacut (1478), chapter 9.
145 See Nallino, Vol. 2, pp. 144-77 and 274-77.

## The Stars of the First Magnitude in the Catalogue of Ptolemy

**Table 21: The Stars of the First Magnitude According to Ptolemy in 137**

| Number | Modern Name | Other name | Reference Toomer | Longitude | Latitude |
|---|---|---|---|---|---|
| 1 | α Boo | Arcturus | V,23 | 177° | +31;30° |
| 2 | α Lyr | Waga | VI,1 | 257;20° | +62° |
| 3 | α Aur | Capella | XII,3 | 55° | +22;30° |
| 4 | α Tau | Aldebaran | XXIII,14 | 42;40° | – 5;10° |
| 5 | α Leo | Regulus | XXVI,8 | 30°;122 | + 0;10° |
| 6 | β Leo | Demebola | XXVI,27 | 30°;144 | +11;50° |
| 7 | α Vir | Spica | XXVII,14 | 40°;176 | – 2° |
| 8 | α Psa | Formalhaut | XXXII,42 | 307° | –20;20° |
| 9 | α Ori | Betelgeuze | XXXV,2 | 62° | – 17° |
| 10 | β Ori | Rigel | XXXV,35 | 49;50° | –31;30° |
| 11 | θ Eri | Acamar | XXXVI,34 | 0;10° | –53;30° |
| 12 | α CMa | Sirius | XXXVIII,1 | 77;40° | –39;10° |
| 13 | α CMi | Procyon | XXXIX,2 | 89;20° | – 16;10° |
| 14 | α Car | Canopus | XL,44 | 77;20° | – 75° |
| 15 | α Cen | Bungala | XLIV,35 | 218;20° | –41;10° |

The longitude of 218;20° of α Cen seems to be a misprint in Toomer and should be 188;20°; thus, Libra 8;20° instead of Scorpius 8;20°.

## The Stars of the First Magnitude in the Catalogue of al-Battānī

**Table 22: The Stars of the First Magnitude According to al-Battānī in 880**

| Number | Modern Name | Other name | Longitude | Latitude |
|---|---|---|---|---|
| 1 | α Boo | As-simak ar-ramih | 10°;188 | +31;30° |
| 2 | α Lyr | An-nasr (capra) | 268;30° | +62° |
| 3 | α Aur | Capella | 66;10° | +22;30° |
| 4 | α Tau | Aldebaran, ad-dabaran | 53;50° | – 5;10° |
| 5 | α Leo | Cor leonis, Qalb al-assad | 134° | + 0;10° |
| 6 | β Leo | As-Sarfah, Dhanab al-assad | 40°;155 | +11;50° |
| 7 | α Vir | Spica, as simak al-azal | 50°;187 | – 2° |
| 8 | α Psa | Fam al-hul al-garnubi | 10°;318 | –20;20° |
| 9 | α Ori | Mankib al-gawza | 73;10° | – 17° |
| 10 | β Ori | Rigl al-gawza | 61° | –31;30° |

| | | | | |
|---|---|---|---|---|
| 11 | θ Eri | Ultima stellarum fluvii | 11;20° | –53;30° |
| 12 | α CMa | Ash-shira al-yamaniyyah | 88;50° | –39;10° |
| 13 | α CMi | Ash-shira ash-shamiyyah | 100;20° | –16;10° |
| 14 | α Car= αNav | Canopus, suhayl al yamani | 88;20° | –75° |
| 15 | α Cen | Centaurus, rigl al-faras | 199;30° | –41;10° |

al-Battānī counted an additional star of magnitude 1 in the constellation of Sagittarius, long: 274°; lat: 6;30°. It is a scribal mistake; this star has the magnitude 5. See Nallino, Vol. 2, p. 163 n. 16.

## The Stars of the First Magnitude in the Catalogue of Abraham bar Hiyya

**Table 23: The Stars of the First Magnitude According to bar Hiyya in 1104**

| Number | Modern Name | Other name | Longitude | Latitude |
|---|---|---|---|---|
| H 1 | α Boo | הנתמך הרומח, סימאך רמך | 30°;191 | +31;30° |
| J 2 | α Lyr | נשר נופל, נתר ואקע | 271;50° | +62° |
| G 3 | α Aur | מושך הרפן, אל עיוק | 69;30° | +22;30° |
| A 4 | α Tau | עין השור והוא אלדברן | 57;10° | – 5;10° |
| L 5 | α Leo | לב הארי, קלב אלאסד | 137;02° | + 0;10° |
| 6 | β Leo | Denebola, Dhanab al-assad | 159° | +11;50° |
| I 7 | α Vir | נתמך לאכחזיו, סמאך אעזל | 14°;191 | – 2° |
| N 8 | α Psa | פי הדג הדרומי | 30°;321 | –20;20° |
| D 9 | α Ori | צד תאומים ימיני, מנתכט גוזא אימן | 81;30° | – 17° |
| B 10 | β Ori | דגל תאומים, דגל אלגיזא | 64;20° | –31;30° |
| C 11 | θ Eri | אחרית הנהר, אכר אל נהר | 14;40° | –53;30° |
| F 12 | α CMa | כלב גדול, שערי עכור | 92;10° | –39;10° |
| E 13 | α CMi | הכלב הקטן, שערי גמיעא | 103;40° | –16;10° |
| N 14 | α Car | כסיל והוא סהיל מהערך ראשון | 91;40° | –75° |
| M 15 | α Cen | דגל הסוס מקדם,דגל אלפרס מקדמה | 202;40° | –41;10° |

For the star α Leo the longitude is 137; 20° in MS Berlin and Malatestiana but 137;02° in MS Paris. This last value is the most likely because of the difference of 14;30° with regard to Ptolemy's list. The star β Leo is not mentioned in bar Hiyya's list; we find instead a star with long: 277;18° and lat: –6°;30' which is unknown. It is certainly the result of a misprint anterior to the two manuscripts considered. In the third Ms Malatestiana, there are additional problems of shift of a column of figures preventing an irrefutable conclusion, but the former coordinates are the most likely. We note also that Abraham bar Hiyya identifies the star כסיל with the star Canopus in the constellation of Argo. Ibn Ezra mentioned the same star in his commentary on Amos 5:8 without any connection with כסיל.

The comparison of al-Battānī's list to that of Ptolemy shows that al-Battānī adds 11;10° to the longitudes of Ptolemy. In fact, this is mentioned explicitly by al-Battānī in Vol. 1, chapter LI.[146] This figure corresponds exactly to a precession of 1.5 degrees in 100 years. Indeed, 880 – 137 = 743 years and 7.43 * 1.5 = 11.15° = 11°;08', which he rounds off to 11;10°. Similarly, the comparison of Abraham bar Hiyya's list to that of Ptolemy shows that bar Hiyya adds 14;30° to the longitudes of Ptolemy. The span of time separating them is 1104 – 137 = 967 years. The precession considered in the construction of this list of stars is then 14.5 / 9.67 = 1.50 degrees in 100 years. This is quite surprising. Abraham bar Hiyya championed a precession of 1 degree in 100 years in both the first part of his book, i.e. the canons of his tables, and in the main tables of the second part of the book, *Luhot ha-Nassi*. Only at the end of these tables did he propose a correction table for those following al-Battānī. He also followed Ptolemy in the last chapter of his book *Tsurat ha-Arets.* Abraham bar Hiyya's position is difficult to understand; it is a real conundrum. Of course, the strict application of his opinion would have reduced the difference with Ptolemy to 9.67° = 9;40°, and his longitudes would have been smaller than al-Battānī's by 11;10° – 9;40° = 1;30°. This could have been embarrassing! However, he had to make a decision and the present situation was certainly unacceptable.

## 11. COMPARISON BETWEEN THE RADICES AND MODERN EVALUATIONS[147]

In order to be able to make a comparison, we note the following elements; see Ajdler (2005):

- al-Battānī mean time + 16.44m = modern mean time.
- We assume that all the tables of al-Battānī and Abraham bar Hiyya are constructed in al-Battānī mean time.
- Although Abraham bar Hiyya ascertained that his tables are constructed for Jerusalem, in fact his radices are derived from al-Battānī and are related to ar-Raqqah time.
- The longitude of ar-Raqqah is 39; 03° = 2.6033h = 2h 36m 20s.

146 Nallino, Vol. 1, p. 124.

147 The modern evaluations are based on the formulas given by Meeus (1991). See chapter 24: "Solar Coordinates"; and chapter 45: "Lunar Coordinates."

- $\Delta T = TD - UT$[148] where TD is the Dynamical time (uniform time) and UT the universal (terrestrial) time. On 21 September 1104 $\Delta T$ = 20m 49s. Therefore, at noon in ar-Raqqah it was 12h – 2h 36m 20s + 16m 26s = 9h 40m 6s UT and 10h 0m 55s TD. The epoch of Abraham bar Hiyya was thus 2124557.91730324 JD in TD.

We have then T= (JD – 2451545) / 36525 = – 8.95241841744.

$L = 280.46645° + 36000.76983 * T + 0.0003032T^2 = 186.535883535° = 186; 32, 9.18°$.

$M = 357.52910° + 35999.05030 * T - 0.0001559T^2 - 0.00000048T^3 = 278.956033624° =$

$= 98; 57, 21.72°$ after subtraction of 180° in order to compare with the ancients who referred to the apogee instead of the perigee.

$L' = 218.3164591° + 481267.88134236 * T - 0.0013268 * T^2 + T^3 / 538841 - T4 / 65194000 = 186.764058997° = 186; 45, 50.61°$.

Elongation = 0 ; 13, 41.43°. The mean conjunction occurred 27m before the epoch instead of 13m 24s.

$M' = 134.9634117° + 477198.8676313 * T + 0.0089970 * T^2 + T^3 / 69699 - T4 / 14712000 = 171.742407839° = 351; 44, 32.67°$ after adding 180°.

$F = 93.2720993° + 483202.0175273 * T - 0.0034029 * T^2 - T^3 / 3526000 + T4 / 863310000 = 24.9463398670°$.

$\Omega = L' - F = 161.817719130° = 161; 49, 3.79°$.

**Table 24: Recapitulative Table**

| Radices at epoch | Abraham bar Hiyya | al-Battānī | Modern estimation |
|---|---|---|---|
| Sun longitude | 187° | 186; 57, 24° | 186; 32, 9.18° |
| Apogee | 75; 30° | 85; 38° | 87; 34,47.46° |
| Sun anomaly | 111; 30° | 101; 19, 24° | 98; 57,21.72° |
| Moon longitude | 187; 6, 48° | 187; 04, 12° | 186; 45,50.61° |
| Elongation | 0; 6, 48° | 0; 6, 48° | 0; 13,41.43° |
| Moon anomaly | 351; 17, 38° | 351; 17, 37° | 351; 44,32.67° |
| Ascending node | 160; 28, 58°[15] | 160; 31, 37° | 161; 49, 3.79° |

The elongation of 0;6,48° leads to a conjunction 13m 24s before the epoch. With an elongation of 0;13,42° the conjunction occurs 27m before the epoch.

148 See Meeus (1991), chapter 9: "Dynamical Time and Universal Time."

In 224 years, the tables of al-Battānī had already lost their exceptional accuracy. A difference of 0; 25,15° in the solar longitude corresponds to a span of time of 10.24 hours!

## 12. ADDITIONAL CONSIDERATIONS

1. R. Abraham ibn Ezra wrote in the complementary text following the tables of R. Abraham bar Hiyya[149]:
   **לעולם עשה תקון השמש כתקון אלבתאני הכתוב בתוך ספר לוחות הנשיא מ״כ, ככה במולד, גם בתקופה שנת העולם, והוצא מקום הכוכבים במהלך השוה בלוחות ההם על דעת אלבתאני אע״פ שהוא אומר שהם לדעת בטולמיוס. ואילו היה כדבריו היו יותר מארבע מעלות חסרים....**
   Thus, ibn Ezra insists that the data ascribed by Abraham bar Hiyya to Ptolemy must in fact depend on al-Battānī. Indeed, Ptolemy's tropical year is 365d 5h 55m 12s, and that of al-Battānī is 365d 46m 24s. The difference is 8m 48s = 8,8m. The distance between Ptolemy and bar Hiyya's epoch is 1104 – 137 = 967 years. The accumulated difference is thus 8509.6m = 5.91 days > 5.7°. Ibn Ezra proves that if the data ascribed to Ptolemy were really his, the difference should amount to at least four days and, in fact, nearly six days. However, Ibn Ezra does not note that the tables of movement are based on Ptolemy, and that it is only in the calculation of the radices that Abraham bar Hiyya felt obliged to "cheat" and adopt radices similar to those resulting from al-Battānī. In any case, Ibn Ezra was the first to note that the tables ascribed to Ptolemy did not follow Ptolemy. In *Sefer ha-Olam*, Ibn Ezra (1937) also wrote the following:
   **והתמה מאדם גדול שתקן לוחות במהלך השוה של אלבתאני והוא אומר כי לוחות של בטלמיוס הם.**[150]
2. Through the former quotation, we see that the name of the tables of Abraham bar Hiyya – **ספר לוחות הנשיא** – was introduced by ibn Ezra.
3. We examined above (Table 9) the problem of the radices ascribed to Ptolemy but that were, in fact, very similar to the radices deduced from the tables of al-Battānī. We saw also that the radices according to al-Battānī were calculated with the highest accuracy. We must therefore exclude the radices according to Ptolemy being calculated with an approximation. In fact, it seems that he

149 Notes based on Ibn Ezra, introduced by a copyist. In MS Paris, the phenomenon is even more marked.

150 Ibn Ezra (1937), p. 10, and Millás Valicrosa (1938), p. 321.

wanted to adopt likely radices but wanted also to make the difference with the values of al-Battānī. In any event, the procedure used is unknown and remains a conundrum.

4. In Ajdler (2005), p. 30, we discussed the last paragraph of chapter 9 of *Sefer Heshbon Mehalekhot ha-Kokhavim* and the corresponding passage of al-Battānī. The problem was to determine whether the tables of al-Battānī were constructed according to al-Battānī's mean time,[151] and the addition of 18' for the mean movement in the longitude of the moon during 31 minutes was necessary in order to be compatible with the mean time of the Handy tables.[152] Israel Eichenstein examined six manuscripts at the Jerusalem Library and found הוסף in all of them. Thus, Abraham bar Hiyya understood that al-Battānī's tables are constructed on the basis of the mean time of al-Battānī, and that we must add 18' to the mean longitude of the moon if we work in the mean time of the Handy Tables. If we assume that Abraham bar Hiyya decided to work in his tables, **לוחות הנשיא**, in the mean time of the Handy tables, we could explain why he did not subtract 27 m between ar-Raqqah and Jerusalem in his calculation. He would have compensated for the 31m between the mean time of the Handy Tables and that of al-Battānī by the 27m between ar-Raqqah and Jerusalem. It is mere assumption, but it would have the huge advantage of explaining why R. Abraham bar Hiyya apparently took no account of the difference of longitude between Jerusalem and ar-Raqqah.
5. We ascertain that R. Abraham bar Hiyya worked systematically with the section of the Arabic calendar of the Tables of al-Battānī.

## 13. CONCLUSIONS

We glanced through the tables of R. Abraham bar Hiyya and examined for that purpose three manuscripts: the manuscript of the Malatestiana library of Cessena, manuscript 1046 of the Bibliothèque Nationale de Paris, and the manuscript of Berlin (MS *Hebräischen Handschriften* n° 649). We wanted to understand the astronomy of R. Abraham bar Hiyya in detail.

At the beginning, after reading the first part of the book, we were persuaded that Abraham bar Hiyya completely followed Ptolemy. This is especially the case

151 Mean time equal to true time on about 11 February.

152 Mean time is equal to true time on about 4 November. Mean time Handy tables = Mean Time al-Battānī+ 31m.

in the important issues of the tropical year, the inclination of the ecliptic on the equator, and the precession of the equinox. In all these cases, Abraham bar Hiyya adopted the point of view of Ptolemy and the ancients.[153]

Later, we ascertained that his tables of the motions of the sun, the moon, and the planets, like his tables of correction for the sun and the moon, were all constructed based on Ptolemy's tables. We noted, however, that the text of the first part of the book, the canon of the tables, rests profoundly on the book of al-Battānī, and includes several literal transcriptions (after translation) of this book. Furthermore, we noted that the radices adopted by Abraham bar Hiyya cannot be justified by Ptolemy's tables. By contrast, they are derived from al-Battānī's tables. Similarly, the tables of correction for the planets are consistent with al-Battānī's similar tables. Note that Abraham bar Hiyya never mentioned the different origin of his radices. Without verification, the reader assumes that the tables, radices included, follow Ptolemy. Similarly, on p. 67 of the printed first part of *Sefer Mehalekhot ha-Kokhavim*, in his explanation of the table of the sun, he refers only to Ptolemy.[154] By contrast, it appears that his list of stars is constructed based on a precession of 1.5 degrees in 100 years,[155] in contradiction to his teaching.[156] This position of Abraham bar Hiyya is surprising and difficult to understand. Our understanding is that, in the short run, he gives precedence to Ptolemy's data of regarding the laws of movement. This is also coherent with the good correlation of Ptolemy's tropical year with the length of the average Jewish year.[157] However, he must accept that the radices calculated with Ptolemy's tables, are, after a span of time of nearly 1000 years, untenable and in contradiction to the current observations. Therefore, at this level, he rests on the tables of al-Battānī. Although he relies

153 By contrast, his younger colleague, R. Abraham ibn Ezra wrote two decades later that the inclination of the ecliptic on the equator is 23°; 35', and that the precession is 1.5° in 100 years. The latter adopted the point of view of al-Battānī and the moderns. See the commentary of ibn Ezra on Amos 5:8, and a critical edition with a supercommentary by Uriel Simon: 'שני פירושי ר' אברהם אבן עזרא לתרי עשר, כרך א' עמ (Bar-Ilan University, 1989), pp. 209-12.

154 He nevertheless takes exception with Ptolemy with regard to the apogee of the sun. Ptolemy considered that it has no movement and always remains at 65°; 30'. Abraham bar Hiyya follows his contradictors and fixes the apogee on the eighth sphere, the sphere of the fixed stars, and gives it a direct movement of 1° in 100 Egyptian years, corresponding to the precession of the equinox.

155 Like al-Battānī.

156 Following Ptolemy.

157 The year of Adda.

more on Ptolemy's tables, he rests on al-Battānī's tables for the calculation of the radices, because these tables are more recent and, for a span of time of about 225 years, they should be more accurate.[158] Notice, however, that his tables of motion, according to Ptolemy, range until 520 years; this is in fact a very long span of time. Furthermore, this attitude challenges his confidence in the exactitude of the Jewish calendar. All these elements are concealed in fact by the introduction of faked Ptolemy radices, which hardly differ from the exact al-Battānī radices.

Toward the end of the tables, Abraham bar Hiyya explains the importance of al-Battānī, "who is considered the greatest astronomer of the Arabic world to such a point that in these countries, he takes precedence over Ptolemy." Finally, Abraham bar Hiyya gives the corrections necessary to obtain the radices according to al-Battānī. We find a few tables allowing for the correction of the first movement tables according to Ptolemy, in order to obtain the movements according to al-Battānī.

Abraham bar Hiyya's tables are thus a mixture of tables and data borrowed from Ptolemy and al-Battānī.[159] This mixture is not very coherent and lacks

158 But, if so, why did he consider astronomical movements for spans of time of 520 years according to Ptolemy?

159 This double dependence was mentioned already superficially and without elaboration in Millás Vallicrosa (1959). The author noted the twofold influence of Ptolemy and al-Battānī and some contradictions. Now, just after the completion of the present paper, we became acquainted with a paper written by Raymond Mercier (2014), published in Stern (2014). This paper attempts to compare the Hebrew manuscripts of לוחות הנשיא, mainly the manuscript 1046 of the Bibliothèque Nationale of Paris, a Latin translation and adaptation extant in a unique manuscript of Cambridge, and a Hebrew version of al-Battānī, and, using a scientific method of deviation curves to establish the period when these tables and others fitted the best. The author also noted the twofold dependence on Ptolemy and al-Battānī, but did not elaborate. He did not examine all the tables systematically, but limited his comparison to a small number of tables. He did not examine the theoretical elements behind each table. He did not notice the contradiction between the radices ascribed to Ptolemy and Ptolemy's tables. Similarly, he did not emphasize the insignificant difference between the radices ascribed to Ptolemy and those ascribed to al-Battānī. These different points are precisely the subject of the present paper. Relating to this Hebrew version of al-Battānī, Mercier wrote that Bar Hiyya might have known this Hebrew version in the early 12th century. If this were the case, I doubt that Abraham bar Hiyya would have written his book *Sefer Heshbon Mehalekhot ha-Kokhavim* and these tables at all. Furthermore, the radices in the tables of this Hebrew version (two manuscripts in the library of Munich already described by Steinscheneider [1895]), were calculated for the epoch 28 February1341. This Hebrew translation thus seems posterior to Abraham bar Hiyya. The author introduces a method of investigation that he calls a deviation curve of a parameter.

explanations and justifications for the choices adopted. From a historical point of view, we may assume that the book, still in manuscript, was popular among educated Jews who had no access to the books of Ptolemy and al-Battānī. The number of extant manuscripts proves it. Nevertheless, the book contains no original elements.

Probably because of this, and also because the study of astronomy disappeared slowly from the curriculum of rabbinical students as early as the 17th century, the book was never printed. Moreover, a correct edition of the astronomical tables would have been a difficult challenge to achieve because of the significant scribal errors[160] in the various manuscripts.

In function of the time, we calculate the value of the parameter in the considered table (for example, al-Battānī), reduced by its modern value. We can, therefore, draw the curve of deviation of this parameter in function of the time and find the moment when the deviation was 1. At that moment, the table of this parameter was exact. This gives us a valuable indication about the period of validity of this table, and probably also the period of its elaboration. Using this method, Mercier shows that the Jewish calendar was in agreement with the solar motion in 930 CE, and that the tables of al-Battānī were in agreement with the values calculated by the modern theories in about 880 CE. The last conclusion is well known; see Ajdler (1996), p. 259. The precision of the experimental true equinox of al-Battānī is famous. The first result is questionable. Indeed, we showed (in Ajdler 2013a, p. 29) that the modern Jewish calendar was perfectly calibrated with regard to the sun in the 243rd cycle. In the 247th cycle, the calibration was very good but still not perfect. In order to understand what it is about, we must recall that the average Jewish year is longer than the tropical year by 6,66m. Today, the accumulated difference is (2015 – 846) * 6.6577 = 7782.85m = 5.40 days. We note that 16 Nissan 4606 AMI fell on 16 April Julian = 20 April, proleptic Gregorian calendar. 16 Nissan 4701 AMI (941 CE) fell on 16 April, Julian = 21 April, proleptic Gregorian calendar. Now 16 Nissan 5803 AMI (2043 CE) will fall on 26 April, outside the limits of the "month of spring." However, coming back to Mercier's deviation curves, we note that this method is questionable and meaningless when dealing, as is here the case, with the tables of Abraham bar Hiyya. These tables were constructed artificially by a non-professional astronomer, without any new observation, based on the extrapolation and rounding off of ancient tables. The tables of Abraham bar Hiyya could not have been better than those of al-Battānī, which were out of date and had lost their precision already at the beginning of the 12th century. In such a situation, when the radices of al-Battānī are mixed with movements of Ptolemy, the results of the method of the curves of deviation depend more on mere chance than on true science, and are meaningless.

160 Scribal mistakes in the copying of Abraham bar Hiyya's manuscripts and additional possible systematic mistakes in all extant manuscripts, i.e. scribal mistakes in the manuscripts at the disposal of Abraham bar Hiyya, such as those of Ptolemy or about Ptolemy and of al-Battānī.

## References

Abraham bar Hiyya (1720). *Tsurat ha Arets*, Offenbach.

Abraham bar Hiyya (1852). *Sefer ha Ibbur*, ed. Filipowski, London.

Abraham bar Hiyya (1969). *Sefer Heshbon Mehalekhot ha-Kokhavim*, ha Shem II.

Abraham ibn Ezra (1937). *Sefer ha-Olam*, ed. Fleischer, J.L. Otsar ha-Hayyim.

Ajdler, J.J. (1996). *Hilkhot Kidush ha Hodesh al pi ha Rambam*. Jerusalem.

Ajdler, J.J. (2005). "The Equation of Time in Ancient Jewish Astronomy," *BDD*, 16: 5-56.

Ajdler, J.J. (2013a). "The Gregorian Revolution of the Jewish Calendar," *BDD*, 27: 17-76.

Ajdler, J.J. (2013b). "The Period of 689,472 Years in the Jewish Calendar and its Applications in Frequency and Probability Problems," *BDD*, 28: 3-44.

Ajdler, J.J. (2015). "Luhot ha-Ibbur II," *BDD*, 30: 7-48.

al-Battānī (1903–08). *Albatanei Opus Astronomicum* in 3 Volumes, ed. C.A. Nallino, Milan.

Baer, Y. (1961). *A History of the Jews in Christian Spain*.

Chabas, J., & B. Goldstein (2009). *Abraham Zacut (1452 – 1515) y la Astronomia en la Pinisula Iberica*. Salamanca.

Chabas, J., & B. Goldstein (2012). *A Survey of European Astronomical Tables in the Late Middle Ages*. Leiden–Boston: Brill.

Cohn, B. (1918). *Der Almanach Perpetuum des Abraham Zacuto, Schriften der Wissenschaftlichen Gesellschaft in Straßburg, 32 Heft*. Straßburg: Karl J. Trübner,.

Danjon, André (1980). *Astronomie Génerale*. Paris.

Delambre, J.B. (1817). *Histoire de l'Astronomie Ancienne*. Paris.

Delmedigo, J. S. (1629). *Sefer Elim*. Amsterdam.

Flamsteed, J. (1672). *De Temporis Aequatione Diatriba*. London.

Foz, Cl. (1998). *Le Traducteur, l'Eglise et le Roi*. Ottawa.

Halma, l'abbé (1813–16). *La Composition Mathématique de Claude Ptolémée* (traduction française) + commentaire de Delambre. Paris.

Halma, l'abbé (1823). *Tables Manuelles Astronomiques de Ptolémée et de Théon*. Paris.

Halma (1825). *Tables Manuelles (Handy Tables)*. Paris.

Hanover, R. Levi (1756). *Tekhunat ha-Shamayim*. Amsterdam.

Lalande, J. de (1792). *Astronomie*. Paris, 3rd edition.

Langermann, Y.T. (1999). *The Jews and the Sciences in the Middle Ages*. Ashgate: Variorum.

Loewinger, Y. (2006). "Ha-Munah beka me-r. Yitshak ha-Yisreli derekh Yashar mi-Kandia ve'ad ha-Hazon Isch." *Ha-Ma'ayan*, 46, 3 (5766).

Meeus, J. (1991). *Astronomical Algorithms.* Richmond, Virginia: Willman Bell.

Mercier, R. (2014) "Astronomical Tables of Abraham bar Hiyya," chapter 8 of Stern (2014).

Nallino, C.A. (1903–05). *al-Battānī sive Albatenii Opus Astronomicum*. Milan.

Millás Vallicrosa, J.M. (1938). "Avodato schel R. Abraham ibn Ezra be-Hokhmat ha-Tekhunah," *Tarbiz*, 9, 3–4 (April–July 1938).

Millás Vallicrosa, J.M. (1947). *El Libro de los fundamentos de las Tablas astronómicas de R Abraham Ibn Ezra*. Madrid–Barcelona.

Millás Vallicrosa, J.M. (1959). *La Obra Séfer Heshbón Mehalekhot ha-Kokhavim (Libro del calculo de los movimientos de los astros)*. Barcelona.

Millás Vallicrosa, J.M. (1949). *Estudios sobre la historia de la sciencia española*. Barcelona, pp. 219-26.

Neugebauer, O. (1975). *History of Ancient Mathematical Astronomy*. Springer.

Pedersen, O. (1974). *A Survey of The Almagest*. Odense.

Rashed, R. (2003). *Histoire des Sciences Arabes*, 3 vols. Paris.

Sela, S (2003). *Ibn Ezra and the Rise of Medieval Hebrew Science*. Leiden–Boston: Brill.

Serfati, G.B. (1968). *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*. Jerusalem: Magnes.

Simon, U. (1989). *Shenei Perushei R. Avraham ibn Ezra li- Trei'assar*. Bar-Ilan University.

Smart, W.H. (1931). *Textbook on Spherical Astronomy*. Cambridge University Press.

Steinschneider, M. (1895). *Die Hebraïschen Handschriften der K. Hof- und Staatsbibliothek in München*. München.

Stern, S. (2014). *Time, Astronomy and Calendar in the Jewish Tradition*. Leiden–Boston: Brill.

Toomer, G.J. (1984). *Ptolemy's Almagest*. Duckworth.

Viete, Fr. (1579). *Canon Mathematicus*. Lutetia (Paris).