# Stability Problems in the Tabernacle

The description of the Tabernacle is one of the most popular subjects of the Torah. Different sections of the Talmud try to deepen and specify the subject and complete the Biblical data. Many books and above all, the commentaries on the Bible, the commentaries on the Talmud and the super-commentaries on Rashi have been devoted to the subject. In the following paper we examine the stability of the Tabernacle under the action of the weight of the curtains and the pulling forces necessary to maintain them and the stability of the Tabernacle under the action of the wind. We demonstrate that the Tabernacle, as it has always been represented, is unstable and must collapse. The stability of the Tabernacle requires a careful guying of the boards of the Tabernacle, in two directions, before and after its erection. Additionally, the curtains require a guying not only to avoid flying away, as noted by the traditional commentaries, but also to pull them and lift them up and give them a form narrowing a horizontal ceiling. The generally accepted scheme of curtains hanging vertically along the walls presents difficulties with regard to the attaching of the guying ties to the boards through the curtains. Therefore it is likely that the curtains hung along an oblique line, parallel to the guying ties of the boards of the Tabernacle and hiding them partially. Similarly, it is likely that oblique ties were visible above the two oblique curtains, guying the two upper curtains which had no springing. The external aspect of the Tabernacle was thus quite different than the generally accepted representation.

We examine also the details of the composition of the bases, the boards of the Tabernacle and the pillars of the courtyard. We show that the bases were esthetical shoes with very thin walls. We show that there are reasons to consider that the boards and pillars were hollow and that the four wagons offered to the Levites of the family of Merari on the inauguration of the Tabernacle were only the first specimens of the required number of wagons allowing the carriage of the dismantled Tabernacle during its journeys.

# Stability Problems in the Tabernacle

The description of the Tabernacle is a very popular subject in the rabbinical literature. This must be related to the importance of its description in the Bible including a repetition at the end of Exodus. Another indication of its continued significance is evident by the fact that the description of the Tabernacle remains a subject studied extensively by Jewish boys following the Orthodox curriculum. The Bible description was completed by the Talmud and Midrashim and by a special Beraïta dealing with this special subject, ברייתא דמלאכת המשכן. We observe further that the number of books dealing with the description of the Tabernacle and its vessel is particularly high, more over that they are a supplement to the traditional commentaries on the Torah and the super-commentaries on Rashi.

Here is a non exhaustive list of descriptions and studies of the Tabernacle:

תבנית ההיכל תבנית המשכן וכליו, ר' יעקב יהודה, אמשטרדם ,ת"י.
Jacob Judah Templo[1] (1603-1675): Retrato del Tabernaculo de Moseh, 1654.
Idem : English translation 1675
חכמת המשכן על המשכן וכליו, ר' יוסף שליט ריקיטי, מנטובה, ת"ם.
מעשה חושב, ר' עמנואל חי ריקי, ויניציאה, תע"ו.
מקדש אהרן, ר' אהרן צבי אבן חן, פטרבורג, תרנ"ד.
המקדש וכליו, ספר מעשה חושב וברייתא דמלאכת המשכן, שאול שפר,ירושלים תשכ"ה.
מלאכת המשכן, משה לוין, תל אביב, תשכ"ט
המשכן וכליו, ר' ישראל חיים בלומנטל, ירושלים תשל"ה
תבנית המשכן וכליו ובגדי הקודש, שלום דוב שטיינברג, תשנ"ה.

Without taking into account all the non Jewish books devoted to the same subject.
It appears that nowadays books are still written about the theoretical description of the Tabernacle and others are more specifically written about the pictorial representation comprising of photos of a reduced model. It is interesting to note that R' Jacob Judah of Amsterdam had already built a reduced model which he offered to Queen Henrietta Maria in 1643 and later a second model that he took with him in England in 1671.
All these descriptions were based on purely geometrical assumptions based on the Biblical descriptions completed by the Talmudic precisions, the opinion of the Rishonim and the classical commentaries and the personal opinions of the authors. We observe that physical examination taking into account the density of silver or gold were never performed although the theoretical knowledge required was already available for a long time. Similarly important stability problems were never taken into consideration. But at

[1] The same as the precedent. Dutch rabbi born in Hamburg, he was educated in Amsterdam, were he was the pupil of R' Isaac Uziel.

this level it must be noted that this particular science began to develop only in the second half of the nineteenth century and mainly in the twentieth century.[2]

1. The Adanim or the Sockets Under the Boards.

The boards of the Tabernacle had a height of 10 cubits and a breadth of 1.5 cubits.[3] The thickness of the boards is not specified in the Torah but it is the object of a discussion in the Talmud.[4] Rashi,[5] Rashbam[6] and all the commentators followed the opinion of Rabbi Nehemia according whom the broads have a constant thickness of 1 cubit. We will follow this understanding.
Under each of the boards of the tabernacle were two sockets of silver for the two tenons of the board.[7] The two tenons of the board were like two pegs or pins projecting from the body of the board. At the bottom of the board, the wood was cut away: half a cubit out of the middle and a quarter cubit all the way around to a height of one cubit,[8] leaving two tenons, each one cubit long with a compass of one half by one quarter cubit.

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[2] I have been objected by an eminent reviewer that our concrete knowledge of the *Mishkan* is to sketchy to allow any quantitative analysis. He advises to make rather a qualitative analysis. First of all the present paper is based on the Talmudic and rabbinic description of the *Mishkan* which is, at the first glance, relatively precise. As a civil engineer, having graduated before the computational revolution (pocket calculators and computers) I am persuaded that it is indispensable to get always plausible orders of size of the forces involved in the studied subject in order to apprehending and solving the problems. This is so often ignored by our young engineers who trust blindly the results of their computer and cannot check the plausibility of the computer calculations. I mistrust qualitative analysis, non supported by calculated orders of size, it may be misleading. Without a simple wind calculation and a calculation of the equilibrium of the weighing hanging yarn it is impossible to understand the scope of the difficulty of the anchoring of the *Mishkan*. It is precisely because such calculations taking aim at the determination of the orders of size were never performed, that even modern scholars did not challenge the accepted representation of the *Mishkan*. Otherwise I thank the reviewer for his important and constructive remarks. In the following the calculations will be exact on the assumption that 1 cubit = 52.4 cm but it must be remembered in the course of the paper that only the orders of size are important. Any precision is illusory; indeed in the case of the bases for example, we have not any objective precision about the tenons, hence the divergent opinions between the commentators. The assumption 1cubit = 52.4 cm seems to be the most likely Talmudic value (see Talmudic Metrology I: The Mile as a unit of Length. Ajdler, J.J, B.D.D. 19, pp. 55 -83). We will afterwards consider a second assumption: a cubit of 45.66 cm according toMaimonides' metrology (see Talmudic Metrology III: Units of Measure of Volume and Capacity, Ajdler, J.J, B.D.D. 21, p. 47); we will see that even if some results differ in the ratio 1: 1.5 the main conclusions of the paper remain valid, independently from the value adopted for the cubit.
[3] Ex. 26 : 16.
[4] B. Sabbath 98b. Rabbi Judah considered a trapezoidal section with a variable thickness of 1 cubit at the bottom and 1 inch (1 breadth of thumb) at the top. See a graphical representation in Humash *Da'at Mikra* Shemot, Vol II: p. 163. Rabbi Nehemia considered a constant thickness of 1 cubit. See a graphical representation in The Tabernacle, Moshe Levine, 1969, pp. 45, 47 and 49.
[5] Ex. 26: 5 and 26: 23.
[6] Ex. 26 : 22.
[7] Ex. 26 : 17. No additional information is given. There are divergent opinions about the details of the tenons: see the opinions of Rashi, Hezkouni and Ramban. These divergent opinions have no incidence on our subject. We follow here the commentary of Rashi, which from the point of view of the strength of materials makes more sense.
[8] See note 10.

Each socket had a compass of three quarters of a cubit by one cubit[9] and a height of one cubit.[10] Its cavity was a hole going right through and had a section of half a cubit by a quarter of a cubit. These dimensions were the same as those of the tenons: the tenons fitted into the cavities of the sockets. The sockets were the foundation of the boards of the Tabernacle; two sockets were the basis of one board.

According to this model the generally accepted idea— and this was indeed the idea that I had always had when I was learning these Biblical sections with my late father —that the basis constituted out of silver, a much heavier material than the wood cut away, conferred a greater stability to the boards of the wall.

It must be noted that even if the foundation had a much greater weight than the wood cut away, the additional stability achieved would be insignificant because the surface of the foundation remained the same as the section of the wall. This would not be the case if the foundation had a breadth of three cubits, for example, instead of one cubit, the thickness of the boards.[11]

In order to go further we must make some assumptions allowing us to cipher data. We will consider a cubit of six palms of 52.4 cm.[12] We will however consider afterwards another assumption of a cubit equal to 45.7 cm and corresponding to the metrology of Maimonides. We will see that despite a difference of 13% the conclusions will remain qualitatively the same. We will further consider that the silver produced at this epoch had a purity of about 80% and had a density of 10.2.[13]
Thus 1 dm3 of silver weighs 10.2 kg and contains 80% fine silver, but the ancients were probably not aware of this lack of purity.
We can now calculate the weight of one silver basis:
Volume: $0.75\ c^3 - 0.125\ c^3 = 0.625\ c^3$.
Volume: $0.625\ c^3 = 90\ dm^3$.
Weight: 918kg.

Now, the Torah tells us that the weight of the silver contained in one basis is one *kikar*, or 3000 Shekels of the Sanctuary. We assume that the Talmudic Sela weighed 14.16 g.[14] and the kikar weighed 3000 * 14.16 = 42.48 kg. Therefore the Shekel of Moses weighed 14.16/1.2 = 11.80 gr, and the kikar weighed 35.4 kg.[15] The weight of the silver in a board

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[9] According all the commentators, two adjacent bases had the same section as one board: 1c * 1.5c, in order not to protrude in the empty space of the Mishkan and reduce its breadth of 10 cubits (see Rashi on Ex. 26: 5 based on B. Sabbath 98b: עשר לעיגרא.
[10] This data is mentioned by Rashi 26: 17 and is accepted by all the commentators. Its origin is B. Sabbath 98b: אמה של אדנים and is an uncontested statement.
[11] See note 9 above.
[12] See Talmudic Metrology I: The Mile as a Unit of Length, Ajdler, J.J, B.D.D 19. 0.524m is the most likely value of the Talmudic cubit. It is based on the assumption that the diagonal of a square of 2000 cubits side is 1 Roman mile of 1481 m.
[13] The density of silver is 10.5 and the density of copper is 8.9. Thus an alloying silver – copper 80%- 20% will have a density of 10.2.
[14] See Talmudic Metrology IV: Halakhic Currency, Ajdler, J.J. B.D.D 22.
[15] See B. Bekhorot 50a : בתר דאוסיפו עילוייהו.It is accepted that the Biblical Shekel was revalued by 20%.The rabbinic weight system is based on this data. However, Rashi (and Ramban in his letter sent from Acco) did

basis was then 35.4 kg. The weight of 35.4 kg of silver is far from the anticipated weight of 918 kg silver. Therefore, the assumptions made by the different commentators are untenable and must be adapted.

Let us consider that each basis has a section of three quarters of a cubit by one cubit and that its walls, including the bottom, have a thickness of *e* measured as a fraction of one cubit. We consider thus a basis in the shape of a prismatic shoe whose four walls and bottom have a thickness of *e*. Thus at the bottom of the board, the wood was cut away:2*e* out of the middle and e all the way round to a height of one cubit minus e leaving two tenons, each, one cubit minus *e*, long with a compass of one cubit minus 2*e* by three quarters of a cubit minus 2*e*. The volume of metal of the shoe is then:

1. Bottom 1 * 0.75 * $e$) $c^3$.
2. Walls 2*(1 * 1* $e$) $c^3$.
3. Walls 2*(1 * 0.75 * $e$) $c^3$.
4. Total 4.25 $e * c^3$

Weight of the basis: $4.25\, e * (5.24)^3 * 10.2 = 35.4$ kg
Thickness of walls and bottom of the basis: $e = 0.06$ dm ~ 6 mm.

On the assumption of a cubit of 45.7 cm[16] we find e ~ 9 mm

The situation is thus completely different from what was expected on the basis of the description of the commentators. It appears that the bases were in fact decorative shoes whose thickness of the walls and the bottom was less than 6 or 9 mm if we accept that the height of the basis was still one cubit. The cutting off of the bottom of the boards was thus insignificant and had only an esthetical motivation. The basis protected the bottom of the boards from the action of water or ground but they had no stabilizing effect; the shape of the bottom of the boards was unrelated to the common representation of a board ending with two little tenons fitting in the cavities of two heavy foundations.[17]

2. The Stability of the Walls.

According to all the representations of the Tabernacle, the stability of the walls is ensured only by their weight, without any guying ties. If we consider a density of the wood of the boards of 0.5, then for one board of $1.5\,c * 1c * 10c = 15c^3 = 2158\ dm^3$on the assumption that c = 52.4 cm, the weight is then 1080 kg.[18]

If a horizontal force $F_h$ is exerted at the top of the board, the torque is $5.24 * F_h$ and the eccentricity is $e = (5.24*F_h)/\ 1.080= 4.856\ F_h$. One can consider that the limit of the

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not take it into account in his commentary on Ex. 21: 32 and 25: 39 and compared the weight of the Shekel to the Uncia of Köln: 1 Sela = 0.5 Uncia of Köln ~ 14.5 gr. Indeed he gives the same weight to the Talmudic Sela in his commentary to B. Bekhorot.

[16] This is the cubit according to Maimonides. It is close to the Royal Egyptian cubit.

[17] See Humash *Da'at Mikra*, Shemot, Mossad ha-Rav Kook, Vol II, p. 153 where the representation of the bases seems in accordance with our conclusion. This representation does however not fit the commentary of *Da'at Mikra*.

[18] Without taking into account the increase in weight because of the replacement of 0.00347 $m^3$ of wood of one board by the silver of the basis.

stability and the beginning of the collapsing is reached when the eccentricity is h/3 where h is the thickness of the boards, i.e. one cubit. If $e$ = h/3, then $e$ = 0.524m / 3 = 0.175 m and $F_h$ = 0.036 T = 36 kg.

In this situation the maximum tension on the ground is σ = 2N/3b (h/2-e) where b is the

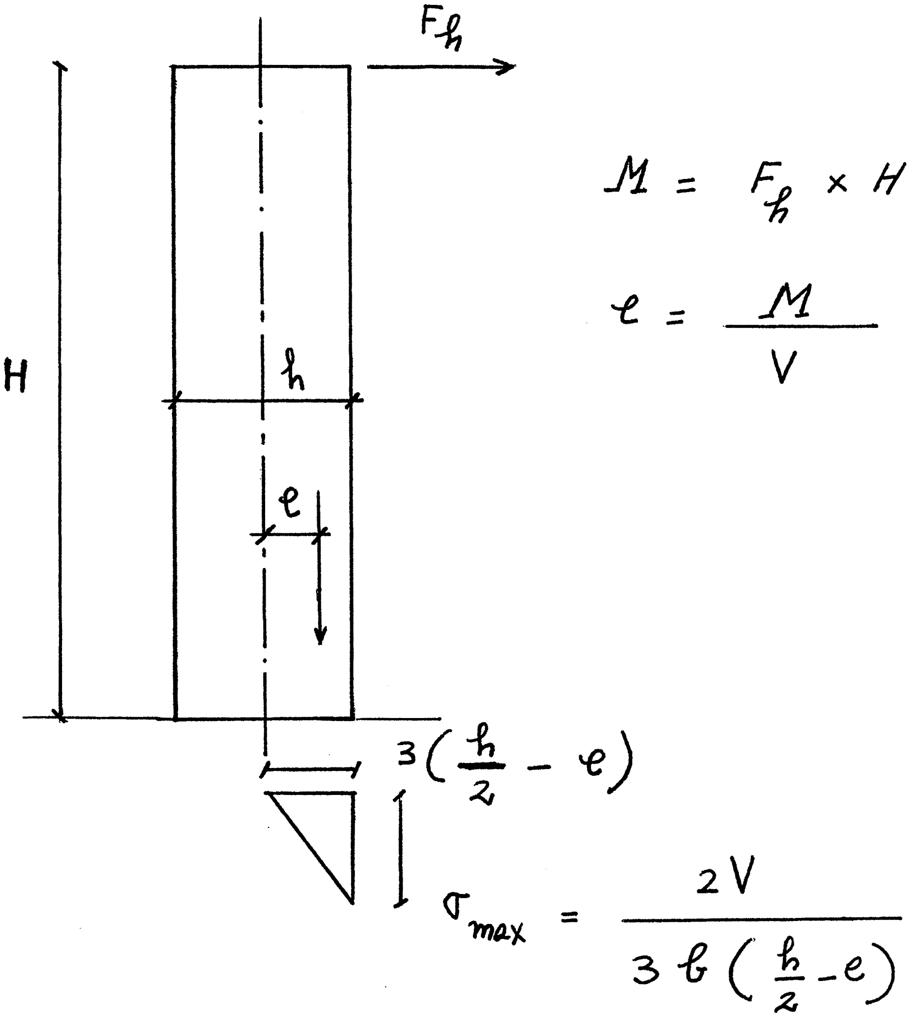



Fig 1. Stability of the wall.

[19] See further. The force is in fact 193 kg/m or 151.70 kg per board.

3. The Curtains Covering the Tabernacle.

The curtains covering the Tabernacle represent a material that is not very common in civil structural engineering. Indeed it has no tightness and can be compared to a weighing yarn. The stability of the ceiling is thus not evident; the study of the stability of systems of weighing yarns is a particular chapter, often neglected, of rational mechanics. The traditional commentaries did not take this problem into consideration. In Midrash Rabbah, Nasso, XII: 18 it states that the curtains around the Tabernacle, hanging along the walls of wooden boards were fixed at their bottom and anchored in the ground by ropes in order to fix them against the action of the wind. Rashi writes about the same.[20] To the best of my knowledge, only one commentator has understood that the fixation of the curtains at the bottom of the walls had a major stability function. Indeed R' Hiskiya ben Manoah in his commentary[21] *Hezkouni* writes that during the phase of the erection of the Tabernacle, the builders were tying rows to the external edges of the curtains along the northern, southern and western walls of the Tabernacle. He explains that they were then pulling the ropes energetically in order to lift the curtains and prevent them from falling in the void of the Tabernacle because of their weight. But when one pulls the rows of the curtains, one must take support on the head of the boards and exert a force, similar to that exerted on the ropes, on the head of the boards. Of course, the boards of the northern and western elevation in the vicinity of the western elevation are rigidified by this western wall but this effect disappears rapidly.

We further assume that the different bars, internal and external, binding together the boards as well as the rings binding the adjacent boards at their top level, are foreseen in order to get a correct alignment of the boards but don't have any stability functions. Similarly, we assume that the little laths of suspension of the curtain at the entrance of the Tabernacle, and of the *Parokhet*, the veil at the entrance of the Holy of Holies, don't participate to the horizontal stability. In these conditions the forces exerted on the head of the walls will cause the reversal of the wall and the collapse of the whole construction if no adequate measure is taken. This problem was never taken into consideration. It is interesting to note that Moses Levine represented the columns of the courtyard maintained with guying ties but he didn't feel the necessity to consider something similar for the walls of the Tabernacle.

Let us examine the problem more thoroughly. In the engineering appendix we examine the equilibrium of a weighing yarn of 2*l length weighing p/m[22] and hanging to two points, *P* and *P'*, distant by 2d from each other and situated on the same horizontal. One demonstrates that the figure of equilibrium of this system is a symmetrical funicular curve passing through *P* and *P'* and having a deflection *f*. The tension prevailing in the yarn, especially at its extremities depends on the deflection *f* on the symmetry axis. The deflection would be zero and the yarn would be perfectly horizontal if the forces exerted at the two extremities were infinite. In the present case the distance 2d is equal to ten

[20] Ex.35:18 and37: 19.
[21] Ex.37: 19.
[22] The weight p per meter of length.

cubits or 5.24 m. We calculate the tension at the extremities of the yarn for different deflections. We consider that there are four layers of curtains, from bottom to top:
--The linen curtains (length: 28 cubits)
--The curtains of the tent (length: 30 cubits).
--Skins of ram.
--Skins of Tahash.[23]

We assume that the weight of each layer is 1.5 $kg/m^2$, the linear weight of the theoretical yarn is thus 6 kg/m if we assimilate the curtains of 1m breadth to a weighing yarn of linear weight p = 6 kg/m.[24]

| Half of the length of the funicular curve: l | Maximum Deflection | Horizontal Component of the Tension in the Yarn |
|---|---|---|
| l = 5.15 cubits | f = 56.4 cm | $T_x$ = 37 kg/m |
| l = 5.05 cubits | f = 32.2 cm | $T_x$ = 64. kg/m |
| l = 5.025 cubits | f = 22.7 cm | $T_x$ = 91 kg/m |
| l = 5.010 cubits | f = 14.4 cm | $T_x$ = 144 kg/m |
| l = 5.005 cubits | f = 10 cm | $T_x$ = 203 kg/m |

The former table gives us a good idea of the evolution of the different parameters of the problem. It shows also the force exerted per linear meter of the walls when pulling the curtains. The lifting of the curtains was thus a dangerous operation: pulling the curtains too much and reducing the deflection induces increasing horizontal forces on the top of the boards, endangering the stability of the walls even if due guying ties were foreseen because of the possibility of easily overstepping the admissible limits.

If we consider that a deflection of 14.4 cm is the maximum acceptable deflection then the tension exerted on the curtains on top of the boards is 144 kg/m[25] and the force exerted

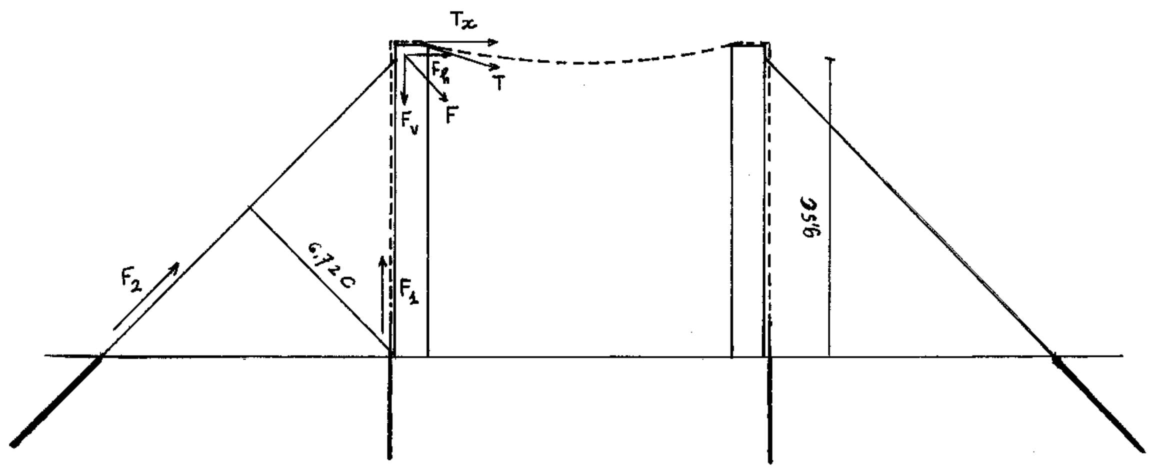
[23] In B. Sabbath 28a, Rabbi Judah says that there were two separate covers above the linen curtains and the curtains of the Tent. Rabbi Nehemia thinks that the skins of the rams and of the Tahash made only one cover.
[24] This weight is arbitrary but it helps calculating an order of size. It was adopted after consulting carpet merchants.
[25] I have been objected: why this value? Why not a deflection of about a meter corresponding to insignificant efforts at the top of the walls? A deflection of 1 m, or even 0.5m, seems esthetically inadmissible. Further we will see later that the stability of the Tabernacle under the action of the wind requires the placement of tie-rods between the two opposite southern and northern walls at such a level warranting that they will never support the curtains. For this reason I adopted for the demonstration, a deflection of 14.4 cm and placed the tie-rods about 25 cm under the top of the boards. There is no reason to diminish to much the tensions because it is the wind which generates the critical efforts. Another reason to limit the deflection is that in the case of storm with depression wind the curtains would undergo an inadmissible movement of flapping. A last reason is that under a thunder storm, which occurs sometimes even in the desert, the weight due to the water, in the case of important deflection, could increase in such a way that the collapse would become unavoidable. Even with a deflection of 14.4 cm the average weight would double, requiring a good security coefficient to avoid collapse. We can also remark that a deflection of 1.02 m corresponds to l = 5.5 c and 2*l = 11 c. In such a case the bottoms of the curtains are raised by 0.5 c ~26 cm. The Hahmei ha-Talmud did not imagine this situation. The discussion in B. Sabbath 98b shows that they considered that the length of the curtains bridging the whole of the Tabernacle was about

on top of the boards requires a special guying of the different boards of the walls. The tensions which are brought into play in the ties are far from negligible and must be balanced by a deep anchoring in the ground or by a stone ballasting.[26]
This last method is less esthetic but could be much more precise and safer because of the possible quantification. Nevertheless in all the cases where these forces have a horizontal component the stone ballasting doesn't fit.

On the assumption of a cubit of 45.7 cm, the deflection of 14.4 cm is reached with l = 5.0133 cubits; it introduces a horizontal component of 109.3 kg/m in the yarn.

1. First Solution: the curtains hang vertically along the wall.[27]

**Figure 2: Section of the Tabernacle on the assumption of hanging curtains along the walls. $F_1$ = 131 kg/m and $F_2$ = 213 kg/m.**




Before beginning to place the curtains on the Tabernacle and to pull and lift them, it was necessary to make the Tabernacle rigid by internal horizontal abutment beams placed between the walls, in order to equilibrate the opposite forces exerted on the tops of the walls during the lifting of the curtains, and to place oblique ties creating an internal wind bracing for equilibrating possible wind action. Under these conditions it was possible to lift the curtains at the desired level and fix the situation by anchoring the ties attached at the bottom of the curtains in the ground.
Before removing the internal wind bracing and the horizontal abutment beams, it was necessary to create an external guying of the construction. One can imagine that these guying ties were attached to rings fixed through a pin going through the width of the board, at its top. These guying ties could equilibrate the forces exerted on the top of the

10 cubits and that the length of the hanging curtains was 8 or 9 cubits for the inferior curtains and 9 or 10 cubits for the superior curtains (Rabbi Nehemia and Rabbi Judah). They figured thus that the curtains were stretched and were close to the horizontal without a significant deflection.
[26] Rashi has already considered the problem in Ex.27: 19.
[27] This is the classical representation of the Tabernacle. See for example the book of Moshe Levine. The necessary forces for counterpoising them are very difficult to mobilize.

boards by the curtains and allow taking away the horizontal abutment beams placed before the beginning of the operation. From the point of view of achieving stability, the horizontal abutment beams were much more efficient but we don't find any element allowing consideration of such abutment beams after the assembling of the Tabernacle. Operating these guying ties poses many difficulties. Indeed the rings, placed on top of the boards, are covered by the curtains and the existence of little slots in the two hanging layers of curtains is the only solution allowing reaching the rings. This is not without a serious problem because the rings are distant by 1.5 cubits, the one from the other, while the breadth of the curtains is 4 cubits and the joins between the inferior curtains fall in the axis of the superior curtains. That means that such a solution requires the creation of little slots corresponding always to the symmetry axis of the boards through the two layers of hanging curtains. The position of the slots would then be different in each curtain of each layer, without any repetition at all. It would then require a very precise positioning and numbering of the curtains during the assembling. The main difficulty, however, was the execution beforehand and with securing precision of the slots.

On the assumption of Maimonides' cubit, the required traction forces are slightly diminished but the problem remains unchanged.[28]
It requires also a temporary internal wind-bracing because the external guying ties can only be attached after the hanging of the curtains.

2. Second Solution: the curtains hang obliquely.[29]

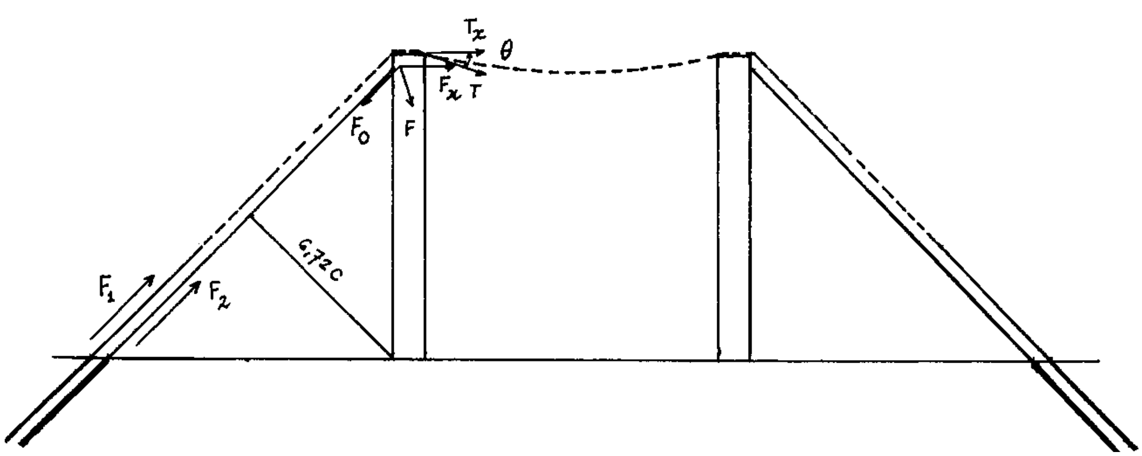



**Figure 3: Section of the Tabernacle on the assumption of curtains hanging obliquely. $F_1$ = 135 kg/m and $F_2$ = 221 kg/m.**

[28] On the assumption of a cubit of 45.7 cm $F_1$ = 97.6 kg/m and $F_2$ = 161.3 kg/m.
We see thus that we don't discuss about a decimal, even with a cubit near to the minimum value possible for a cubit we observe that the stability of the work requires traction forces difficult to counterpoise.
[29] See Humash Da'at Mikra, Shemot, Vol II, pp. 145 and 181 where a similar solution is represented.

A second solution can be considered without the problem of the slots in the curtains and the difficulty of their implementation. In this case, the external guying ties considered above must now be placed as soon as the walls are erected, before any operation of placing the curtains on the boards or beginning to lift them. This means that it will not be possible to place the curtains afterwards, according to the traditional scheme, with the curtains hanging along the walls of the Tabernacle. The only possible solution is to place the overstepping of the curtains, with regard to the ceiling of the Tabernacle, along an oblique line parallel to the guying ties of the boards. The ties which are placed at the bottom of the curtains in order to anchor them will also be parallel to the guying ties of the boards. This second solution contradicts the generally[30] accepted scheme of curtains hanging vertically along the walls.[31] Nevertheless the Biblical text is not decisive, as the verb סרח means more "to stretch," than "to hang". Another advantage of this disposition would be to justify why the external face of the boards were covered by a golden sheet and the external bars were also covered by a gold envelope.[32] The chosen slope of 45° is not the most elegant but it diminishes the anchorage forces.

On the assumption of Maimonides' cubit, the required traction forces are slightly diminished but the problem remains unchanged.[33]

4. Wind-bracing of the Tabernacle.

Besides the problem of the lifting of the curtains, which is at the origin of important forces acting on the top of the boards and can provoke the collapse of the Tabernacle, the problem of the wind bracing must also be considered. The effect of the wind on the wall facing the wind is similar to the effect of the pulling force on the curtains; but the effect of the wind on the wall under the wind[34] is opposed to the former effect and corresponds to forces directed toward the exterior of the Tabernacle: it will tip the walls towards the exterior. The great difficulty of a building like the Tabernacle is the absence of a solid roof. It creates problems that engineers never encounter in solid buildings. In order to

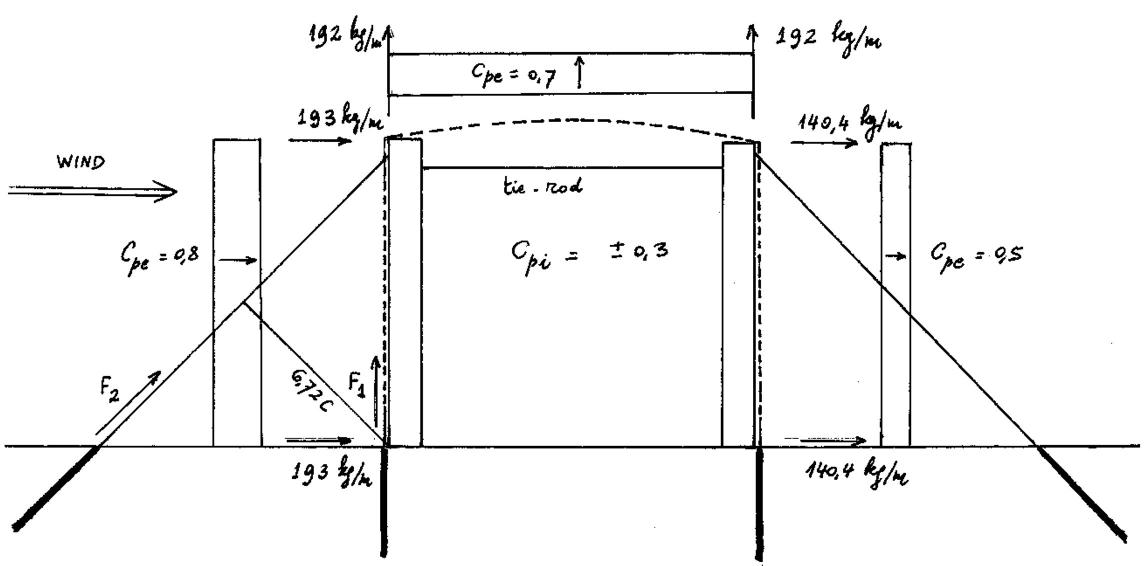
[30] See however note 29.
[31] In the edition of the Torah by Mossad ha-Rav Kook with the commentary Da'at Mikra, it represents the Tabernacle, according to a representation credited to architect Jacob Judah (the same name as the rabbinical author of the seventeenth century Jacob Judah Templo) with curtains which don't hang along the walls but are pulled along an oblique direction. However the stability of the walls was not considered. The calculation in B. Sabbath 98b of the length of the exposed part of the boards, not covered by the curtains, according to Rabbi Nehemiah and Judah (contradiction about the thickness of the top of the boards) could lend credence to the generally accepted idea that the curtains were hanging vertically along the walls. But it is also possible that these calculations are related to the special situation when the curtains were stretched vertically along the walls; but this was not the normal position of the curtains.
[32] The details of the golden covering of the boards are not given in the Bible. In Ma'asseh-Hosheev (edition Shafar pp. 43-45) it examines the problem. It appears that there are different opinions on the subject, whether the covering was thin or thicker, whether it covered all the surface of the boards or only the visible surfaces or only the top of the boards.
[33] On the assumption of a cubit of 45.7 cm $F_1$ = 97.6 kg/m and $F_2$ = 161.3 kg/m.
We see thus that we don't discuss about a decimal, even with a cubit near to the minimum value possible for a cubit we observe that the stability of the work requires traction forces difficult to counterpoise.
[34] Sheltered from the wind.

avoid this form of collapse we should bind the two opposite walls together with horizontal tie-rods binding the tops of the opposite boards.

If we consider a wind according to the Eurocode,[35] blowing from North to South, the characteristic[36] wind acting on the northern wall, facing the wind represents (0.8 + 0.3) * 67 kg/m² ~ 74 kg/m². This evenly distributed load can be replaced by two forces of 193 kg/m acting on the top and foot of the wall. This force acting on top of the wall is still greater than the force considered above under the weight of the curtains. The wind acting on the southern wall, under the wind, represents (0.5 + 0.3) * 67 kg/m² ~ 54 kg/m² and is equivalent to two forces of 140 kg/m applied on top and on the foot of the southern wall. The effect of the wind on the roof is a depression force of (0.7 + 0.3) * 67 kg/m² = 67 kg/m$^2$ equivalent, after subtraction of the weight of the roof of 6 kg/m$^2$ to two vertical forces of 192 kg/m acting at the extremities of the roof.

1. First Solution: the curtains hang vertically along the wall.

**Figure 4: The curtains are hanging vertically along the walls. $F_1$ = 178 kg/m and $F_2$ = 496 kg/m.**




[35] New European standards, aimed to uniform the building standards through Europe.
[36] There is only a 5% chance that the wind is greater than the characteristic wind during a year. Nevertheless during a period of 50 years the exceptional wind is 130% of the characteristic wind. In some applications, such as the verification of an anchor in the ground of guying ties or traction piles, the exceptional wind must be taken into consideration.

In the first solution the curtains hang vertically along the walls, the bottom curtains hanging by eight cubits and the second layer of curtains hanging by nine cubits along the walls.[37] The southern wall or the wall under the wind is submitted to a wind-force acting on top of the boards directed to the exterior of the Tabernacle of 140 kg/m. The great problem is to decide whether we can rely on the curtains to equilibrate this force and transmit it to the top of the northern wall, the wall facing the wind. The problem is the following: we have decided that the deflection of the curtains should be 14.4 cm corresponding to 2*l = 10.02 cubits. This means that the top of the southern wall could move to the south under the wind by 0.02 cubits or 1 cm. But the adjusting of the curtains is not precise and this movement would be dependent on the precision of the adjustment. Furthermore, the boards have an infinite rigidity and the movement of the top of the boards is the result of the rotation of the foot of the board. It is important that the movement of the soil remains elastic allowing a recovery to the vertical when the wind is no longer blowing.
For all these reasons such a hazardous situation is unacceptable and we should impose tie-rods between the tops of the opposite walls. These tie-rods must be placed after the placing of the curtains, about 25 centimeters under the top of the boards at such a level warranting that the tie-rod must never support the weight of the curtains. These tie-rods must also be considered under the general guying accessories described in Ex.35: 18 and 37: 19. It appears that the traction forces to counterpoise are very important.[38]
On the assumption of Maimonides' cubit, the required traction forces are slightly diminished but the problem remains unchanged.[39]

2. Second solution: the curtains hang obliquely.

We are facing the same problems as in the first solution but the necessity of the tie-rods between the tops of the opposite boards is even greater. As already mentioned, the great difficulty in ensuring the stability of the Tabernacle is that in the absence of a hard roof we must be concerned with the stability of both walls.
It is certain that the oblique parts of the curtains were also subject to the depression forces of the wind which still increases the traction on the anchoring piles. It appears that the traction forces to counterpoise are very important.[40]

On the assumption of Maimonides' cubit, the required traction forces are slightly diminished but the problem remains unchanged.[41]

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[37] Under the assumption that the boards had a uniform thickness of one cubit according to the opinion of Rabbi Nehemia in B; Sabbath 98b. There is a second opinion of Rabbi Judah according to which the thickness of the boards was one cubit at the bottom, narrowing towards the top to a finger's breadth.
[38] As we see on Fig. 4 the traction forces are $F_1$ = 178 kg/m and $F_2$ = 496 kg/m.
The necessary forces for counterpoising them are very difficult to mobilize.
[39] On the assumption of a cubit of 45.7 cm $F_1$ = 155.5 kg/m and $F_2$ = 432 kg/m.
We see thus that we don't discuss about a decimal, even with a cubit near to the minimum value possible for a cubit we observe that the stability of the work requires traction forces difficult to counterpoise.
[40] As we see on Fig. 5 the traction forces are $F_1$ = 265 kg/m and $F_2$ = 496 kg/m.
The necessary forces for counterpoising them are very difficult to mobilize.
[41] On the assumption of a cubit of 45.7 cm $F_1$ = 231 kg/m and $F_2$ = 432 kg/m.
We see thus that we don't discuss about a decimal, even with a cubit near to the minimum value possible for a cubit we observe that the stability of the work requires traction forces difficult to counterpoise.

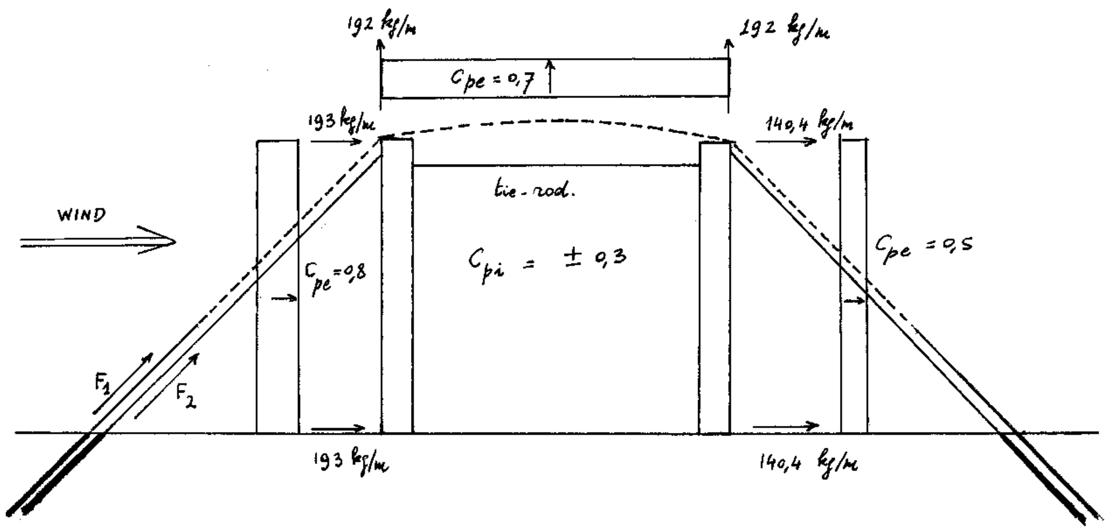
5. Additional Problems.

1. The boards.

The boards have the dimensions of 1.5 c * c * 10 c or 2.16 m³ according to the assumption of one cubit equal to 0.524m.[42] The problem is similar with a shorter cubit. The question is whether this was a board of massive wood, plywood or a mortise-and-tenon joint providing an internal empty space. A board of massive wood

of such a dimension raises many problems. Was it possible to find in Egypt trunks of

**Figure 5: The curtains are hanging obliquely. $F_1$= 265 kg/m and $F_2$ = 496 kg/m.**




trees allowing shaping boards of such a dimension? Another problem is technological: were they able to treat such a board of massive wood and dry it without causing significant cracks? Even today it would be practically impossible to guarantee the good behavior of such a board of massive wood. Another problem is the weight of such a board representing about 1.08 T (ton).[43] The Levites of the sons of Merari where in charge of the heavy carriage but they disposed only of four wagons while the Levites of the sons of Gershom disposed of two wagons.

[42] See note12.
[43] On the assumption of Maimonides' cubit the weight of one board is 0.71 T.

On the assumption of boards and pillars in massive wood the total weight of items under the responsibility of Merari could approach 132 T,[44] bases, board, pillars and ropes and equipment included. This seems however impossible for the following reasons:

-- Resistance of the wagons.
-- Volume available for storage on the wagons.
-- Resistance of the axles of the wagons.
-- Pressure on the ground under the wheels.
-- Traction capacity of the bullocks.

All these elements are complementary and confirm that this is impossible. In particular we know that today the strongest draught-horses are those bred in Perche (France) and in Ardennes (Belgium). They had a wide reputation and were exported world wide during the nineteenth century. Under ideal conditions, such horses can pull a load of about one ton and draught-bullocks would pull a similar load. Therefore, we should consider a wagon of 2 T weight corresponding to a maximum useful storage of about 1.6 T.[45]

Therefore, I would champion the idea that these boards and pillars were shaped with mortise-and-tenon joints providing an internal empty space, lightening considerably these works. The boards could be constituted with boards of 2 cm thickness assembled with mortise and tenons, with stiffeners and braces and even massive tympana at the top of the boards allowing placing the top rings, in the middle of the height, placing the inner bar and at the bottom, and placing the additional boards in order to constitute the tenons put

[44] They must carry one hundred bases of silver weighing, according to our assumption, 3.54T and sixty bases of the pillars of the courtyard weighing about 2.1T. They must carry sixty pillars of the courtyard, forty-eight boards of the Tabernacle, four pillars of the *Parokhet* and all the guying ropes. If the forty-eight boards were in massive wood, they would represent about 51.8 T (on the assumption of Maimonides' cubit it would be 34.3 T). Similarly, if we consider that the pillars were of massive wood and had the dimensions of c * c * 15c representing a volume of 15 c3 or 2.158m$^3$, their weight would be 1.08 T like a board of the Tabernacle and the sixty pillars would weight 64.74 T (but only 21.6 T if we consider columns of 5 cubits. On the assumption of Maimonides' cubit 64.74 T would reduce to 42.8 T). We must still add the weight of four bases and four pillars of the *Parokhet*, of five bases and five pillars of the entrance of the Tabernacle. Thus the weight carried by the four wagons could reach 3.54 + 2.1 +0.23 + 51.8 + 64.74 + 6.48 = 128.83 T ( 87.26 T on the assumption of Maimonides' cubit), without taking into account the accessories and the guying ropes. The total weight of the equipment under the charge of the tribe of Merari would then be about 132 T.
Note that there are divergent opinions about the height of the hangings and the pillars of the courtyard. Rabbi Yosse thinks that the hangings and the pillars had a height of 15 cubits while Rabbi Judah, according to the literality of the text Ex. XXVII: 19 considers that the height of the courtyard was only 5 cubits (see B. Zebahim 59b-60a and also B. Erubin 2b). However, from B. Sabbath 98a- 99a, it seems that the longest pieces had a length of 10 cubits and the model of wagons described in the Talmud would not have allowed carrying pillars of 15 cubits length. The commentary Hezkouni, ad locum, considers that the hangings had a height of 5 cubits above the bases and the pillars had six cubits. In the Humash with the commentary *Da'at Mikra* they represented a courtyard with 5 cubits height, hangings and pillars. David Levine has represented the courtyard with pillars of 15 cubits height. All the representations show pillars with a square section of 1cubit size but R' Emanuel Hay Ricci (see Ma'asseh Hosheev, ed. Shafar p. 84) champions a circular section of one cubit diameter.
[45] Considering a tare of 0.4 T.

in the sockets. I estimate the weight of such a board to 250 kg.[46] Similarly the weight of the pillars of the courtyard would weight 250 kg. Therefore the weight of the heavy equipment of the Tabernacle would be about 35.09 T[47] without taking into account the accessories and the guying ropes. This brings us to a load of about 38 T or 9.5 T per wagon. This is still too much and it would be cautious to consider the necessity of at least 38 / 1.6 or twenty- four wagons.[48]

This conclusion doesn't seem to be embarrassing. Indeed the princes had offered at the inauguration of the Tabernacle six wagons and Moses gave two of them to the tribe of Gershom and four to the tribe of Merari but it is never written that these four wagons were sufficient and represented the total number of wagons. In fact, the princes were the first to think about the problem of the displacement of the Tabernacle and their offering was probably the occasion of becoming aware of the logistic problems and organizing another "national subscription" to offer the supplementary required wagons as soon as Itamar[49] and his logistics staff had evaluated the needs.

Another way to apprehend the problem is the following: the weight of the boards and the pillars, according to our assumption that the volumes of the boards and pillars are hollow, was about 250 kg. If we place 4 boards[50] or 4 pillars[51] on a wagon and complete the load with bases, ropes and other accessories up to 1.6 T, we will need (1/4) *117 or 29 wagons.
Thus the most likely number of necessary wagons for the tribe of Merari is between 24 and 29 wagons. On the assumption of a cubit of 45.7 cm (Maimonides) these figures would be diminished but the problem would remain.

In order to get a better idea of the weight carried by a wagon, we will also examine the weight of the curtains carried in the two wagons of the sons of Gershon. The load was constituted by the linen curtains,[52] the curtains of the Tent[53] the skins of rams[54] and the

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[46] Another solution would be to consider massive boards with a littler thickness; for example a thickness of 12 cm giving a similar weight. This would correspond to the understanding of Ibn Ezra on Ex. 26: 18. But this assumption creates two difficulties: -- this model removes us away from the Talmudic model-- the bases must be broader than the little thickness of the boards of about 12 cm. – However the weight of the basis is limited to 35.4 kg and this forbids to have a structural basis and limits it to an esthetic shoe.

[47] 3.54T + 2.1 T +.23 T + 12 T + 15 T + 2.22 T = ~ 35 T

[48] On the assumption of Maimonides' cubit this figure should be re-examined and diminished but the principle remains the same.

[49] See Ex.38: 21 and Num.4: 28.

[50] In the book Mikdash Aharon it writes that when the Tabernacle was dismantled, the bar-rings were taken odfd to avoid damaging the gold overlay of the boards. The connecting pins were also removed and the boards were placed on the wagons one next to the other and one above the other.

[51] In B. Sabbath 98a and 99a it deals with the wagons and the storage of the boards. The wagons are described as having a length of 5 cubits and an (internal) width of 2.5 cubits, but a bulkiness of 5 cubits (thickness of the lateral walls, wheels and space between wheels and lateral walls included). The Talmud considers that one places four boards on their thickness on the wagon. Rashi concludes that one placed 3 layers of 4 boards per wagon; the only way to carry the 48 boards. But he pays no attention to the pillars. Similarly he does not compare the pulling load of the two draught-bullocks and the load of the wagon loaded with 12 boards.

[52] $28 * 40 * (0.524)^2 * 1.5 = 461.29$ kg.

[53] $30 * 44 * (0.524)^2 * 1.5 = 543.66$ kg.

skins of Takhash[55] and their ropes. It comprised also the hangings[56] of the courtyard and their ropes. It contained also the screen of the entrance of the Tabernacle[57] and the veil of the Holy of Holies.[58] This gives, under our assumption of 1.5 kg/m2 for the curtains and 0.75 kg/m2 for the hanging of the courtyard, a weight 2188 kg without the weight of the accessories and ropes. The total must thus narrow 3.2 T; this confirms again our assumption that the weight of a wagon, useful load plus tare, could not overstep 2T. We see thus that our assumption of a maximum load of 2T per wagon and 1 T per bulk is likely. This confirms our assumption that the two wagons that were given to the sons of Gershom were exactly sufficient for their use.

On the contrary, the four wagons given to the sons of Merari did not meet their requirements and were only the first specimens of the required wagons.

2. The Gold Covering of the Boards

Let us consider that the curtains were hanging along the walls vertically according to our first assumption. This fits the generally accepted representation of the Tabernacle by the rabbis.  One can then ask oneself why the external surface of the boards, which remains completely unseen, the bars, and the bar-rings were covered with gold. This is better suited to the second solution where the external surface of the first boards was visible.

3. The two upper curtains

According to the accepted opinion that the upper curtains didn't hang along the walls, how were these curtains attached to the underlying curtains in order not to fly away under the action of the wind? Undoubtedly they must be held by guying ties which must be visible on the hanging curtains whether they hanged vertically or in oblique.

4. The wind force acting on the bottom of the boards

The horizontal wind force acting on the bottom of the boards was regained by the friction of the boards on the ground. Indeed the horizontal force is about 200 kg/m and the weight of the board is about 1.08 *T* corresponding to 1.37 *T*/m. But now that we have established with much likeliness that the boards were probably hollow and weighed only about 250 kg corresponding to 318 kg/m we face a new problem: how to equilibrate these forces. One possibility would be to accept the existence of a wooden floor bound to the bases, but there is no allusion to the existence of such a floor. Otherwise the only solution would be to accept that the bases of the board were partially buried in order to mobilize thrust.

5. The Stability of the Western Wall

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[54] $12 * 32 * (0.524)^2 * 1.5 = 118$ kg.
[55] $12 * 32 * (0.524)^2 * 1.5 = 118$ kg.
[56] $280 * 15 * (0.524)^2 * 0.75 = 864.91$ kg.
[57] $10 * 10 * (0.524)^2 * 1.5 = 41.19$ kg.
[58] $10 * 10 * (0.524)^2 * 1.5 = 41.19$ kg.

The stability of this wall under the depressive effect of a wind, blowing from east to west, creates a problem because the external guying ties cannot work in compression. The problem has two possible solutions: whether the different bars of the wall and the top-rings give to this short wall enough stiffness to support this wind and report the reactions on the two adjacent walls; or, they were obliged to create two tie-rods bound on two rings fixed at the top of the two middle boards of the western wall. These two tie-rods were bound to the rings of the tie-rods binding the fourth boards of the northern and southern walls. They played the same role as the tie-rods binding the opposite boards of the northern and southern walls.

6. Conclusions

We have examined some problems related to the Tabernacle in the spirit of civil engineering, taking into account physical elements like the weight of the materials, the forces exerted by the curtains, and the action of the wind. We have seen that these elements lead us to conclusions which are far from the generally accepted ideas. The stability problems of the Tabernacle are serious problems and cannot be considered frivolous. The erecting team had the responsibility to master stability concepts and quantify the involved forces in order to avoid experiencing collapse during the phases of the erection or under the storm of wind. We have seen that these phenomena are difficult to quantify and subdue. Taking into account Maimonides' cubit of 45.7 cm instead of the more likely Talmudic cubit of 52.4 cm has only a little influence and does not change the problematic at all.
While throughout history, authors have described the Tabernacle in extraordinary detail, these problems were completely ignored, and none ever imagined the necessity of guying the walls of the Tabernacle in addition to the guying of the curtains.

We have demonstrated that stability requirements oblige us to consider additional guying-ropes and even horizon tie-rods under the ceiling of the *Mishkan*. We are even obliged to change our minds about the exact nature of the boards of the Tabernacle. Finally we have examined the logistical aspects of the displacement of the Tabernacle and shown that the Tribe of Merari needed much more than the four wagons offered by the Princes of Israel at the occasion of the inauguration of the Tabernacle.

## Engineering Supplement[59]

We consider a weighing yarn hanging between $P$ and $P'$, distant by 2d from each other. We consider coordinate axes *Ox* and *Oy*. The distance on the axis *Oy* between *O* and *A*, the point of the yarn situated on the axis *Oy*, is called a. The equilibrium figure of the yarn has the equation $y = a * \cosh(d/a)$ (1)
This equation corresponds to the equation of a funicular curve. The length of the curve counted from *A*, point of intersection of the curve with the *Oy* axis until a point of abscises $x$ is given by $s = a * \sinh(x/a)$. (2)
Let us assume that the length of the yarn is 2l.

[59] See P. Gaudiot : Cours de Mécanique Rationnelle, tome II : cinématique et dynamique des systèmes. pp. 321-323. Paris Eyrolles 1947.

From (2) we deduce: $l = a * sinh$ (d/a). (3)

We have also: $f = a * [cosh ((d/a) – 1)]$ (4)

$T1 = a* p$ (5)

$T2 = a * p * cosh$ (d/a) (6)

We know l and $d$, we must find $a$ in order to calculate $f$, $T1$ and $T2$.
There is no analytical solution to the problem. We can nevertheless develop *sinh* and *cosh* in series: $sinh\ x = x + x^3/3! + x^5/5! + x^7/7! + \ldots\ldots$

$$cosh\ x = 1 + x^2/2! + x^4/4! + x^6/6! + \ldots\ldots$$

If we take only the two first terms then (3) becomes $l = a * (d/a + d^3/6a^3)$

$$l = d + d^3/6a^2$$

Hence $\boldsymbol{a^2 = d^3 / 6 (l-d)}$. (7)

Similarly, (4) becomes: $f = a * [1 + d^2/2a^2 -1] = d^2/2a$. (8)

Thus $f^2 = d^4/4a^2 = (d^2/4) * 6(l-d)/d$.

Hence $\boldsymbol{f^2 = 1.5 * d*(l-d)}$. (9)

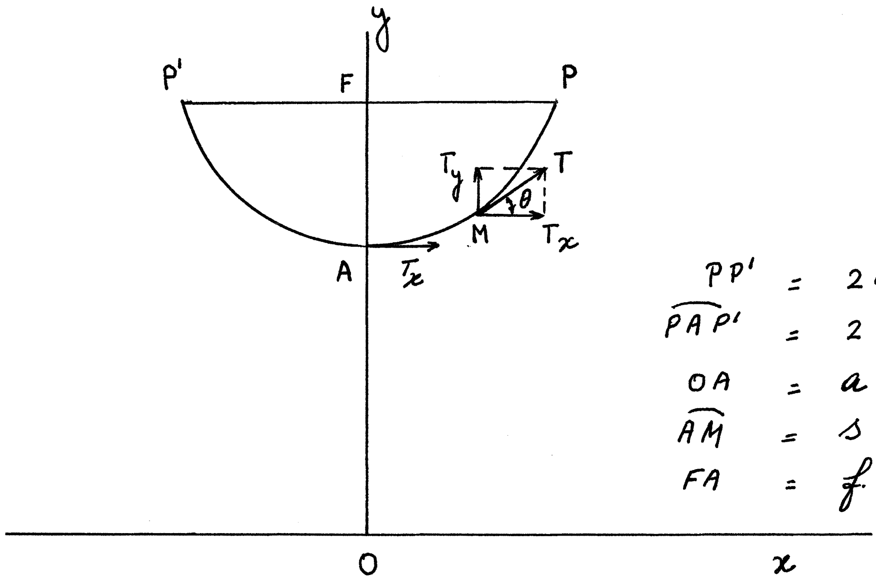
Remark. From (8) we deduce: $d^2 = 2af$ and more generally $x^2 = 2ay'$ or $y' = (1/2a) * x^2$.
We see that the approximation made by taking only the two first terms of the series corresponds to replace the curve $y = a * cosh (x/a)$ by $y = a * [1 + x^2/2a^2]$ or $y' = (y – a) = x^2/2a^2$. The approximation consists of replacing the funicular curve by a parabola of parameter a.

Example.

If $d = 5c$ and $l = 5.025c$, then $a = 28.87c = 15.13m$. (**7**)

The horizontal component of the tension force $T_x$ is a constant
$p*a = 6 * 15.13 = 90.76$ kg/m. (**5**)

The deflection is $f = 0.43c = 22.69\ cm$. (**8**)




$$PP' = 2d$$
$$\widehat{PAP'} = 2\ell$$
$$OA = a$$
$$\widehat{AM} = s$$
$$FA = f$$

$$y = a \cosh \frac{x}{a} \qquad\qquad f = a\left(\cosh \frac{d}{a} - 1\right)$$

$$\operatorname{tg} \theta = y' = \sinh \frac{x}{a}$$

$$\frac{1}{\cos \theta} = \cosh \frac{x}{a}$$

$$T_x = p.a = C$$
$$T_y = p.s = pa \sinh \frac{x}{a}$$
$$T = p.y = pa \cosh \frac{x}{a}$$

$$a = d \sqrt{\frac{d}{6(\ell - d)}}$$

$$f = d \sqrt{\frac{3(\ell - d)}{2d}}$$

Table 1