How did Maimonides choose his epoch? Explanations about the astronomical calculations in H.K.H.[1]

In B.D.D. 26 Ariel Cohen published an article in Hebrew with the preceding title.
In B.D.D. 27 I reacted and the review published finally my two series of critical remarks and Cohen's answers.
The aim of this paper is to try reestablishing the scientific truth and helping disoriented readers to understand better what it is going on.

1. Maimonides' epoch

Ariel Cohen chose arbitrary and without any justification the epoch of Maimonides on Wednesday evening, the beginning of Nisan 3rd, 4738 at 6 p.m. MoJMT = Modern Jerusalem Mean Time =17:39 h IST (Israel Standard Time) = 3h: 39m UT. In fact we noted already that the assumption to fix the epoch at 6 p.m. = MoJMT is unfunded and even anachronistic. Indeed the modern mean time was defined only in 1673 when Flamsteed, the astronomer Royal, published the modern mean time. Before Flamsteed the mean time was calibrated differently and coincided with the true time either on November 4 or on February 11.[2] In the first case the correction from true time to mean time is always additive, it corresponds to the choice of Ptolemy in his Handy Tables; it corresponds also to the "equations des horloges" as it was tabulated in the almanac "La Connoissance des Temps" during the eighteenth century.[3] As the correction from true time to mean time is always additive, the mean time occurs always before the true time.[4]
In the second case, the correction from true time to mean time is always subtractive; this corresponds to the choice of Ptolemy in the Almagest and of al-Battani in his astronomical book. It is also certainly the system adopted by Maimonides. As the correction from true time to mean time is always subtractive, the mean time occurs always after the true time.[5]
In our times (21th cent, in the English speaking world – till now no official worldwide sign convention) we define Equation of Time (EoT) = the additive correction applied to mean time T_M of the clocks to get the apparent (= true) time T_A, of the sundials, so that EoT = $T_A - T_M$ (French textbooks apply a different sign convention where their equation of time is – EoT = $T_A - T_M$).

Ariel Cohen justified his assumption of Maimonides' epoch at 6 p.m. Jerusalem (modern) mean time by referring to the astronomical commentary of Otto Neugebauer: Notes to chapters XI to XIX, p. 128 in Yale Judaica Series, Vol XI published 1956 and reprinted in 1967. In fact the astronomical notes of Neugebauer represent a simplified and popular study for the understanding by laymen of Maimonides' text. His magisterial work was a study published in Huca in 1949, Vol. XXII: "The Astronomy of Maimonides and its Sources" pp. 321 – 363. In this study Neugebauer assumed at the beginning that the epoch was at 0 h[6] or 6 p.m.[7] However, at the end of his study, he was obliged to conclude that the epoch was rather at 6h 23m p.m. which he rounded off

[1] I thank Engineer Ya'akov Loewinger, who read the paper and checked the calculations and the astronomical definitions.
[2] These dates evolve very slowly with the centuries.
[3] During the eighteenth century the "Connoissance des temps" published both, the "équation des horologes" according to the ancient conception of the equation of time and the table of the equation of time according to the new definition of Flamsteed.
[4] At noon true time, it is already after noon in mean time, thus mean noon occurred before true noon.
[5] At noon true time, it is still before noon in mean time, thus mean noon occurred after true noon.
[6] Standard Jewish time measured from 6 p.m.
[7] Jerusalem mean time. At that time, in 1949, Neugebauer had not yet written his magisterial book about the history of mathematical astronomy and he was probably still unaware of the difference of 17.57 minutes between the mean time of Ptolemy and our modern mean time and of a difference of 16.44minutes between the mean time of al-Battani and our modern mean time. At that time, Neugebauer did not make any difference between modern and ancient mean time. When he wrote the astronomical commentary in 1956, he probably did not pay attention to the problem of the equation of time and did not consider the difference between the mean time of the ancients and the moderns. Therefore, he maintained his initial assumption of an epoch at 6 p.m. without drawing the attention of the reader on the difference between the mean time of the ancients and of the moderns. It is only in his monumental "History of Ancient Mathematical Astronomy" 3 Vol. Springer Verlag, 1975 that he considered comprehensively the problem of the equation of time. I introduced these elements in a paper in B.D.D. 16 and could draw important conclusions about the epoch of Maimonides.

to 6h 20m p.m. His conclusion was therefore that it is more accurate to ascertain that the epoch was chosen to be at 0h 20m (Jewish time) of Nisan 3 of the year 4938 A.M. He connected this value with the remark made by Maimonides, that the moment of vision is 20m after sunset.
In fact Maimonides writes that the epoch is at the beginning of the night.[8] He writes also that the moment of vision is about 20 minutes after sunset.[9] But he defines also somewhere else the beginning of the night as 20m after sunset.[10] Finally from the beginning of chapter 12, we understand that the addition of complete days, counted from the epoch leads always, after correction for the inequalities of the days during the year to the moment of vision. Therefore it seems that the epoch was itself a moment of vision; thus 20 minutes after sunset. The conclusion of Neugebauer would thus be in agreement with the conclusion of Maimonides' text: the epoch is at 6h 20m p.m; it corresponds to 20m after sunset. Indeed near to the equinox, sunset is at about 6 p.m.

2. Our solution.

We know that the epoch was at the beginning of the Jewish Thursday 3 Nisan 4938 = on Wednesday 22 March 1178 at the beginning of the evening, at the moment of the first visibility of the Nisan New Moon Crescent. This possibility of visibility on the moon on the night of the epoch, near the true vernal equinox (on 13/3/1178) was the reason of choosing this date as epoch. This enables to find the data for next evenings of vision, by adding values for round (29 or 30) days to the radices.
I don't understand why the fist vision close after the true vernal equinox was important for Rambam.

The radices of Maimonides are the following:

Mean sun's longitude:	7°; 03', 32" = 7.0589°
Mean moon's longitude	31°; 14', 43" = 31.2453°
Sun's apogee	86°; 45', 08"
Mean anomaly	84°; 28', 42"
Ascending node	– 180°; 57', 28"

We can deduce that the mean elongation at the epoch was 24.1864°.

Now from the data given by Maimonides for 10000 days, we know that the mean movement of the sun per day is 0°; 59', 08.33" = 0.98564777 °/d. The mean movement of the moon per day is 13°; 10', 35.03" = 13.17639722°/d. The relative movement is then 12.19075 °/d.

The mean conjunction occurred 24.1864 / 12.19075 = 1.9839960625 d before the epoch or (2d – 414.82 hal) before the epoch. We will thus know the epoch with precision if we can find the moment of the mean conjunction. We have the possibility to find the mean conjunction in the book of al-Battani on two ways: whether according to his tables in Roman years or according to his tables in Egyptian years.

A. **Roman years.**
Al-Battani counts his Roman years[11] according to the era of the Bicornute[12] (Alexander the Great).[13] The epoch of this era is 0 March – 310, about 6 months after the beginning of Maimonides' era of Contracts, which begins in Tishri 3450 AM.[14] Thus the considered mean conjunction of 20 March 1178 must correspond in al-Battani 's tables to the conjunction of Adar[15] 1489 S.E.[16] Referring to the conjunction tables we find:

[8] Hilkhot Kiddush ha-Hodesh or H.K.H. 11:16 12:2.
[9] H.K.H. 14:6.
[10] Hilkhot terumot 7:2.
[11] Julian years of 365.25 days.
[12] Dhu'l quarnaym.
[13] Or S.E. = Seleucid Era.
[14] The year 3450 AM corresponds to the Julian years –311 and –310. It begins in September – 311.

1479	10d 30' 24"	355°; 50' 30"	Vol 2, p. 84
10	9 47 37	9 ; 15' 17"	Vol 2, p. 86

1489	20d 18' 1"	365°; 5' 47"	

Thus the first conjunction of the Roman year 1489 of the era of Dhu'l quarnaym was on Adar (March) 20 at 18' 1" or 7h 12m 24s p.m.[17] al-Battani mean time in ar-Raqqah.

The epoch was thus on 22 March at 7h 12m 24s – 23m 3s[18] or 22 March at 6h 49m p.m. expressed in al-Battani time, al-Battani mean time calculated in ar-Raqqah =AlBRMT.
The epoch was thus on 22 March at 6h 49m – 27 m[19] or 22 March at 6h 22m p.m. expressed in 'al-Battani mean time in Jerusalem' AlB JMT.

B. **Egyptian years.**

The Egyptian years are normally counted from the era of Nabonassar, on 26 February – 746 at noon corresponding to JD 1448638.[20] Al-Battani counts his Egyptian years[21] from the era of Dhu'l quarnaym (Alexander the Great): Egypt. Year 1 of Dhu'l quarnaym = Year 437 of Nabonassar; there is always a difference of 436 years between the years of these two eras. The day of the mean conjunction considered is 20 March 1178 = JD 2151401. The epoch of Nabonassar is 26 February – 746 = JD 1448638. The day of the conjunction was thus the day 2151401 – 1448638 + 1 = 702764 of the era of Nabonassar = 1925 * 365 + 139; thus the 139th day of the year 1926 of Nabonassar or the 139th day of the 1490th Egyptian year of the era of Dhu'l quarnaym. We are thus looking for the conjunction of the fifth month of the 1490th Egyptian year of the era of Dhu'l quarnaym. Referring to the conjunction table we find:

1490	21d 10' 41"	248°; 39' 54"	pg. 29
4months	118 7 21	116 25 39	pg. 31

	139d 18 2	365°; 5' 33"	
5th month	19d 18' 2"	5°; 5' 33"	

Maimonides considered that Nisan 4938 or Nisan 1489 Shetarot, which corresponds to March 1178, corresponds to Adar 1489 Dhu'l quarnaym. Thus the the year 1489 Dhu'l quarnaym began six months after the year 1489 Shetarot. Indeed if it had begun 6 months before then we would already be in the year 1480 Dhu'l quarnaym. We could object that this proves only that Maimonides considered that the beginning of the era of Dhu'l quarnaym was 6 months after his era of the contracts. Another proof of more weight results from the comparison of an eclipse observed by al-Battani with the description of the same eclipse in modern almanacs.
Al-Battani writes in Vol I p. 56 that he observed a solar eclipse in ar-Raqqah on August 8, 1202 Dhu'l quarnayn
[15] In his Roman tables Adar = March and Nisan = April .at 1h 7.5m p.m. Mucke-Meeus give for this eclipse the date of April 8, 891. Therefore:
August 8, 891 = August 8, 1202 Dhu'l quarnayn
August 8, – 310 = August 8, 1 Dhu'l quarnayn and finally
March 1, – 310 = March 1, 1 Dhu'l quarnayn., indeed 6 months after the beginning of Maimonides' Era of Contracts.
[16] 1178 + 311 = 1489. More precisely 1 Dhu'l quarnayn = – 310. 1489 Dhu'l quarnayn = – 310 + 1488 = 1178.
[17] The day is divided in 60' or 3600". 18' 1" = 1081 /3600 = 0.300277 * 24 = 7.206666 h = 7h 12m 24s.
[18] 414.82 halakim correspond to 23.05m.
[19] According to al-Battani the longitude of Jerusalem is 66°;30' and the longitude of ar-Raqqah is 73° ; 15'.
The difference of longitude is thus 6°; 45' or 27 m.
[20] At the moment of the era, 1448638 days of the Julian period had elapsed.
[21] Years of 365 days.

Thus the conjunction of the 5th month, the month of Tybi, of the Egyptian year 1490 of the era of Dhu'l quarnaym was on Tybi 19th, at 7h 12m 48s.We note a good correspondence between the results of these two tables, the moment of the mean conjunction and the common longitude of the mean sun and moon.

We can conclude that the epoch was on 22 March at 6h 22m p.m. expressed in al-Battani mean time in Jerusalem.

3. Calculation of Maimonides' radices from the tables of al-Battani (pg. 72 – 76).

The epoch of Maimonides is 6h 49m p.m. expressed in al-Battani time, al-Battani mean time calculated for ar-Raqqah, as represented in Al.B. tables, Nallino II p. 72 , for SE 1471= CE 1160

Time	Mean Sun	Mean Moon	Moon anomaly	Ascending node
1471 SE	345°; 25'13"	235°; 58'02"	304°; 07'49"	191°; 40'28"
18 y	359°; 40'29"	221°; 38'42"	209°; 11'50"	348°; 05'58"
22d	21°; 41'03"	289°; 52'51"	287°; 25'47"	1°; 09'55"
6h 49m	0°; 16'48"	3°; 44'33"	3°; 42'38"	0°; 0'55'
Radices of Al.B. at epoch of Maimonides	7°; 3'33"	31°; 14' 8"	84°; 28'04"	–180°; 57'16"
Maimonides	7°; 3'32"	31°; 14'43"	84°; 28'42"	–180°; 57'28"

The correspondence between the values of al-Battani and Maimonides' radices is thus outstanding but it is not perfect. This could be the result of an imprecision in the interpolation necessary to calculate the effect of the span of time of 6h 49m. We note that the mean coordinates of the sun and moon at the epoch are, according to al-Battani: sun: 7.05916666 and moon: 31.2355555555. The mean elongation is then: 24.17638888.
The mean conjunction was thus 24.17638888 / 12.19075 = 1.98317485708 d = (2d – 436.11hal).
The mean conjunction of al-Battani was thus on 20 March at 6h 49m + 24.23m = 7h 13.23m p.m. in perfect agreement with the next calculation.

4. Calculation of the mean conjunction from the tables of al-Battani (pg. 72 – 76).

We have seen that the mean conjunction occurred on 20 March at about 7h 12.5m p.m. In fact we find the conjunction at 7h 13.2m.

Time	Mean Sun	Mean Moon
1471 SE	345°; 25'13"	235°; 58'02"
18 y	359°; 40'29"	221°; 38'42"
20d	19°; 42'46"	263°; 31'41"
7h 13.2m	0°; 17'47"	3°; 7'50"
conjunction	5°; 6'15"	5°; 6'15"

There is thus a little difference of maximum1 minute between the different tables of al-Battani. It seems likely that Maimonides used the first table of conjunction; it is the easiest to use. We note also the marvelous precision and coherence of the building of al-Battani.

5. The time of the epoch in Jerusalem in different time scales

The epoch of Maimonides was thus on 22 March at 6h 49m p.m. expressed in 'al-Battani mean time in ar-Raqqah' AlB RMT and it was on 22 March at 6h 49m – 27 m = **22 March at 6h 22m p.m.** expressed in al-Battani mean time in Jerusalem AlB JMT. At the epoch the mean longitude of the sun was 7°; 03'32" and the true longitude of the sun was 9°; 00' 17". In the table of the equation of the days called in Nallino's translation of al-Battani:[22] *Aequatio Nychtemeron*, we find for a true longitude of 9° in the sign of Aries 2°; 57' corresponding to 11.80m.[23] For al-Battani the correction from mean time to apparent (true) time is always additive, 6h:22 min + 11.80 min = 6h : 33.8 min p.m. Therefore the epoch occurred at 6h 33.8m p.m which we will round off to **6h 34m p.m** = the time of vision of the Nisan moon Jerusalem Apparent (=true) Time = JAT.

Because of the relation *Mean Time of al-Battani* (AlB JMT) + *16.44m = Modern Mean Time* (Mo JMT), we can also calculate the modern mean time in Jerusalem **Mo JMT of the epoch**: 6h 22m p.m. + 16.44 **= 6h 38** m p.m. rounded off to 6h 38p.m.

The modern value of the equation of time is given by **EoT = l – α**, with $T_M = T_A$ - EoT where α is the modern true right ascension 7.746° and l is the modern mean longitude =. 6.746° (both values calculated with the program Kiddush 2005, ver.20 of Tsekoni). We find EoT= -1.23° corresponding to -5 m. Thus **Mo JMT** = 6h 34 m p.m.(JAT). – (– 5 m) (EoT)= **6h 39** m. We see that theory of the equation of time of the ancients gives the similar results as that of the moderns.

The true JAT time corresponding to the epoch is thus 6h 34 m p.m. As the epoch is itself a moment of vision, it should be about 20 m after sunset. Sunset would then occur at 6h 14 m p.m. true JAT time. We check that on the day of the epoch the sun reached a depression of about 1 degree at about 6h 14m,[24] this moment is very near to apparent sunset, the moment when the upper limb of the sun disappears at the horizon (6h 12 min). As the exact value of the refraction was not yet known it is likely that Maimonides considered that 1° represent a good approximation of the sum of the sun's semi-diameter and the effect of the refraction (of which he could not be aware quantitatively).

---

[22] Nallini C.A. (1903 – 05) al-Battani : Opus Astronomicum, Milano.

[23] See Nallino, book 2, pg. 62.

[24] The declination of the moon is given by: $\sin \delta = \cos \beta \sin \varepsilon \sin \lambda$. With ε = 23°; 35', λ = 9°; 00' 17" and $\beta = 0$. We find δ = 3.5902°.
The hour angle at a depression of 1° is given by $\cos H = (\cos 91° - \sin \varphi * \sin \delta) / (\cos \varphi * \cos \delta)$. We find H = 93.4298° corresponding to 6h 13m 43s ~ 6h 14m p.m.